# Polynomial Sufficient Conditions of Well-Behavedness for Weighted Join-Free and Choice-Free Systems

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# Outline

# Objectives

Designing well-behaved embedded systems, ensuring

- the sustainability of all their functionalities (liveness)
- bounded memory (boundedness)

efficiently (in polynomial time).

We focus on models of applications that

- allow interesting expressiveness
- do not need to be checked through simulations.

# Outline

**What is new ?** (informal)

- we emphasize some relevant models of embedded systems
- these models have already been partially studied in the past :
  - a characterization of the "good structures" already exists (can be checked in polynomial time)
  - an algorithm for designing systems with a "good behaviour" exists (ILP, exponential time)
- we provide the first polynomial sufficient conditions that ensure the good behaviour of these systems
- we highlight general, very simple and powerful graph transformations

# Outline

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2. Petri nets : definitions and properties
   - Weighted and ordinary nets
   - Special classes of nets
   - Markings and firing sequences
   - Liveness and boundedness

# Weighted and ordinary nets

A (weighted) net is a triple $N = (P, T, W)$ where :

- the sets $P$ and $T$ are finite and disjoint, $T$ contains only transitions and $P$ only places,
- $W : (P \times T) \cup (T \times P) \mapsto \mathbb{N}$ is a positive function.

$P \cup T$ is the set of the elements of the net.

An arc is present from a place $p$ to a transition $t$ (resp. a transition $t$ to a place $p$) if $W(p,t) > 0$ (resp. $W(t,p) > 0$).

An ordinary net is a weighted net in which $W$ is valued in $\{0, 1\}$.

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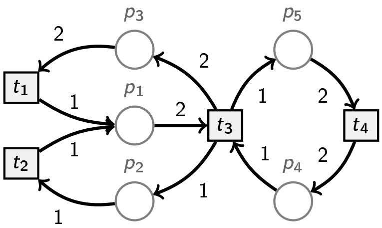
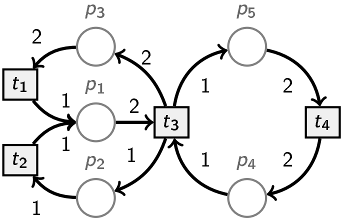


# Weighted and ordinary nets

The incidence matrix of a net $N = (P, T, W)$ is a place-transition matrix $C$ defined as

$$\forall p \in P, \forall t \in T, C[p, t] = W(t, p) - W(p, t)$$

where the weight of any non-existing arc is 0.




$$\begin{array}{c|cccc} & t_1 & t_2 & t_3 & t_4 \\ \hline p_1 & \multicolumn{4}{c}{} \\ p_2 & \\ p_3 & \\ p_4 & \\ p_5 & \end{array}$$

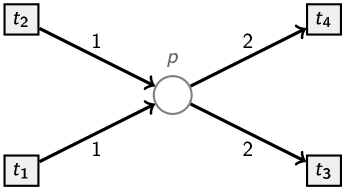
$$\begin{array}{c c} & \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array} & \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -1 & 1 & 0 \\ -2 & 0 & 2 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \end{array}$$

A weighted net and the corresponding incidence matrix.

# Weighted and ordinary nets

The pre-set of the element $x$ of $P \cup T$ is the set $\{w | W(w,x) > 0\}$, denoted by ${}^\bullet x$.
By extension, for any subset $E$ of $P$ or $T$, ${}^\bullet E = \bigcup_{x \in E} {}^\bullet x$.

The post-set of the element $x$ of $P \cup T$ is the set $\{y | W(x,y) > 0\}$, denoted by $x^\bullet$. Similarly, $E^\bullet = \bigcup_{x \in E} x^\bullet$.

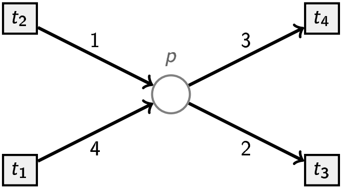



The pre-set of $p$ is $\{t_1, t_2\}$. The post-set of $p$ is $\{t_3, t_4\}$.

# Weighted and ordinary nets

$max_p$ is the maximum output weight of $p$.

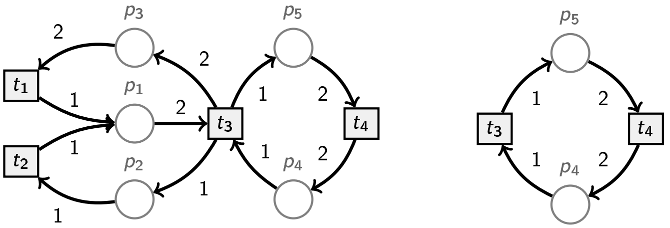
$gcd_p$ is the greatest common divisor of all input and output weights of $p$.




$max_p$ is 3, $gcd_p$ is 1.

# Weighted and ordinary nets

A P-subnet $S = (P', T', W')$ of a net $N = (P, T, W)$ is generated by a subset of places $P' \subseteq P$ and is such that $T' = {}^\bullet P' \cup P'^\bullet$. $W'$ is the restriction of $W$ to $P'$ and $T'$.



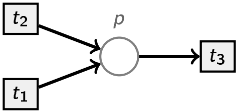

On the right, a P-subnet of the net on the left, defined by the set of places $\{p_4, p_5\}$.

# Outline

2. Petri nets : definitions and properties
   - Weighted and ordinary nets
   - Special classes of nets
   - Markings and firing sequences
   - Liveness and boundedness

# Special classes of nets

$N = (P, T, W)$ is a (weighted) Choice-Free net if any place has at most one output transition, i.e. $\forall p \in P$, $|p^\bullet| \leq 1$.




A T-net is a Choice-Free net such that any place has at most one input transition, i.e. $\forall p \in P$, $|{}^\bullet p| \leq 1$.

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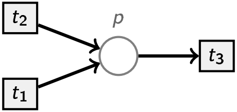


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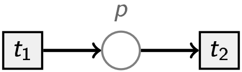

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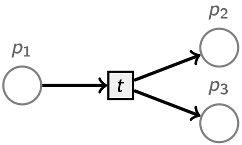


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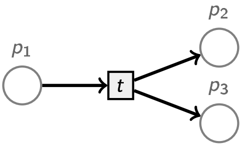


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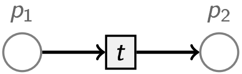
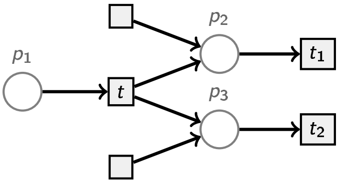


# Special classes of nets

A Fork-Attribution net (FA) is both a Join-Free and a Choice-Free net.




- T-nets are not included in FA nets
  (a transition of a T-net may have several input places)
- S-nets are not included in FA nets
  (a place of an S-net may have several output transitions)

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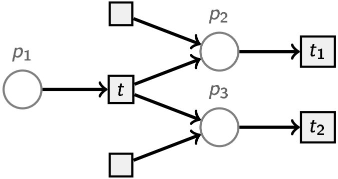


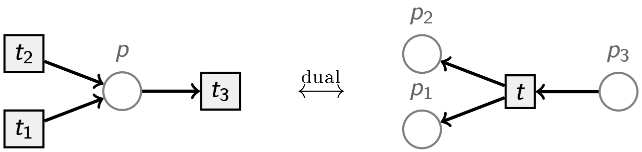

- T-nets are not included in FA nets
  (a transition of a T-net may have several input places)
- S-nets are not included in FA nets
  (a place of an S-net may have several output transitions)

# Special classes of nets

The dual of a net is defined by reversing the arcs and swapping places and transitions.

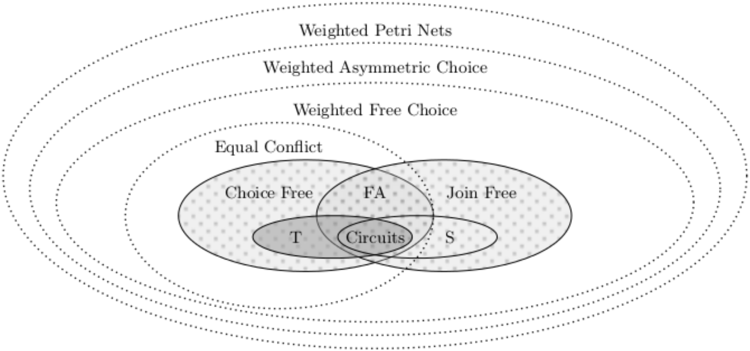
- ▷ amounts to transposing the incidence matrix.

- Choice-Free and Join-Free classes are dual
- S and T classes are dual




Transforming a net into its dual is often not sufficient to deduce behavioral properties of one net from the other.

# Special classes of nets




Some subclasses of weighted Petri nets.

# Outline

2. Petri nets : definitions and properties
   - Weighted and ordinary nets
   - Special classes of nets
   - Markings and firing sequences
   - Liveness and boundedness

# Markings and firing sequences

A marking $M$ of a net $N$ is a mapping $M : P \rightarrow \mathbb{N}$.

A system is a couple $(N, M_0)$ where $N$ is a net and $M_0$ the initial marking of $N$.

A marking $M$ of a net $N$ enables a transition $t \in T$ if

$$\forall p \in {}^{\bullet}t, M(p) \geq W(p, t) \, .$$

A marking $M$ enables a place $p \in P$ if $M$ enables all its output transitions.

The marking $M'$ obtained from $M$ by the firing of an enabled transition $t$ is defined by $\forall p \in P, M'(p) = M(p) - W(p, t) + W(t, p)$

- we note $M \xrightarrow{t} M'$.

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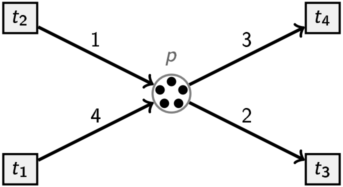
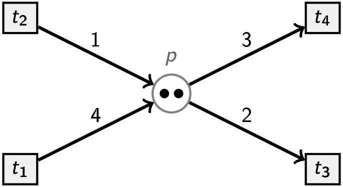
# Markings and firing sequences




The place $p$ is enabled.




The transition $t_3$ is enabled but the place $p$ is not enabled.

# Markings and firing sequences

A firing sequence $\sigma$ of length $n \geq 1$ on the set of transitions $T$ is a mapping $\{1, \ldots, n\} \to T$.

A sequence is infinite if its domain is countably infinite.

A firing sequence $\sigma = t_1 t_2 \cdots t_n$ is feasible if the successive markings obtained $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \cdots \xrightarrow{t_n} M_n$ are such that, for any $i \in \{1, \cdots, n\}$, $M_{i-1}$ enables the transition $t_i$.

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A marking $M'$ is said to be reachable from the marking $M$ if there exists a feasible firing sequence $\sigma$ such that $M \xrightarrow{\sigma} M'$.

The set of reachable markings from $M$ is denoted by $[M\rangle$.

The Parikh vector $\vec{\sigma} : T \to \mathbb{N}$ associated with a finite sequence of transitions $\sigma$ maps every transition $t$ of $T$ to the number of occurrences of $t$ in $\sigma$.

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   - Special classes of nets
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   - Liveness and boundedness

# Liveness and Boundedness

## Liveness

A system $S$ is live if for every marking $M$ in $[M_0\rangle$ and for every transition $t$, there exists a marking $M'$ in $[M\rangle$ enabling $t$.

## Boundedness

$S$ is bounded if there exists an integer $k$ such that the number of tokens in each place never exceeds $k$. Formally,

$$\exists k \in \mathbb{N}\ \forall M \in [M_0\rangle\ \forall p \in P,\ \ M(p) \leq k\,.$$

$S$ is $k$-bounded if, for any place $p \in T$,

$$k \geq \max\{M(p) | M \in [M_0\rangle\}\,.$$

## Well-behavedness

A system $S$ is well-behaved if it is live and bounded.

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## Well-behavedness

A system $S$ is well-behaved if it is live and bounded.

# Structure of a net

### Structural liveness

A net $N$ is structurally live if there exists a marking $M_0$ such that the system $S = (N, M_0)$ is live.

### Structural boundedness

A net $N$ is structurally bounded if the system $S = (N, M_0)$ is bounded for every $M_0$.

### Well-formedness

A net is well-formed if it is structurally live and structurally bounded.

# Consistency and conservativeness

$\mathbb{1}^n$ is the vector of size $n$ whose components are all equal to 1.

### Consistency

A net $N$ with incidence matrix $C$ is consistent if there exists a vector $X \in \mathbb{N}^{|T|}$ such that $X \geq \mathbb{1}^{|T|}$ and $CX = 0$.

### Conservativeness

A net $N$ with incidence matrix $C$ is conservative if there exists a vector $Y \in \mathbb{N}^{|P|}$ such that $Y \geq \mathbb{1}^{|P|}$ and ${}^tYC = 0$.

# Characterization of well-formedness

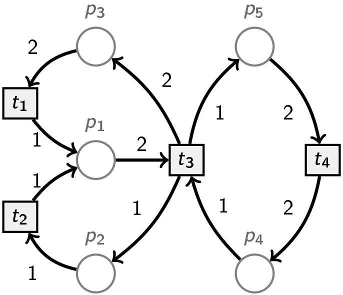
**Theorem (Teruel, Colom, Silva 97)**

Suppose that $N$ is a (weighted) Join-Free or Choice-Free net.
The properties

1. $N$ is consistent and conservative
2. $N$ is well-formed

are equivalent. Moreover, any connected and well-formed Join-Free or Choice-Free net is strongly connected.

# Characterization of well-formedness




$$\begin{pmatrix} 2 & 2 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -1 & 1 & 0 \\ -2 & 0 & 2 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} = {}^t\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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The weighted Choice-Free net is both consistent (right vector ${}^t(2, 2, 2, 1)$) and conservative (left vector $(2, 2, 1, 1, 1)$), thus well-formed.

# Outline

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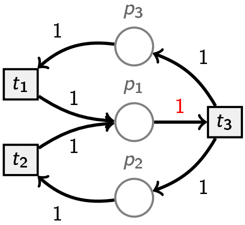
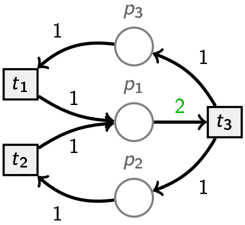
3 Relevance of the study
- Weights are useful
- Expressiveness
- Relevant properties

# Weights are useful

They permit to consider nets whose structure with ordinary weights would be discarded :




An ordinary and unbounded
Fork-Attribution net
(poorly designed)




A weighted and well-formed
Fork-Attribution net
(well-designed)

# Outline

3 Relevance of the study
- Weights are useful
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# Expressiveness

- Choice-Free nets allow
  - several processes to write in the same buffer
  - each process to read and write several buffers
- Join-Free nets allow
  - several processes to read and write in the same buffer
  - each process to write several buffers
- Weights provide a lot of flexibility in the design

# Outline

3. Relevance of the study
   - Weights are useful
   - Expressiveness
   - Relevant properties

# Relevant properties

- Well-formedness is a structural characterization of the well-designed nets
- Well-behavedness is a behavioral characterization of the well-designed systems

▷ Both can be guaranteed in polynomial time ! ◁

# Outline

# Outline

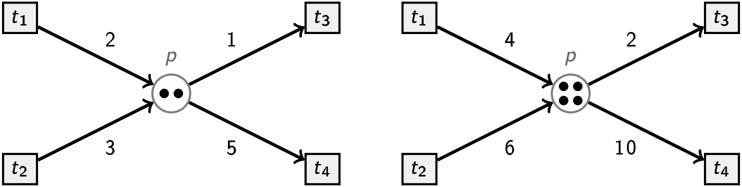
4 Results

- Polynomial transformations preserving the sequences of firings
  - Scaling
  - Balancing
  - Useful tokens
- Well-behavedness of Join-Free systems
- Well-behavedness of Choice-Free systems
- Sufficient conditions are not necessary

# Scaling

## Definition

The multiplication of all input and output weights of a marked place $p$ together with its marking by a strictly positive rational $y$ is the scaling of the place $p$ if the resulting input and output weights and marking are integers. If each place $p$ of a system is scaled by the component $Y[p]$ of a vector $Y$, the system is said to be scaled by Y.




The place on the left is scaled by 2, yielding the place on the right.

# Scaling

**Theorem**

*Let $S = ((P, T, W), M_0)$ be a system and $Y$ a vector of $|P|$ strictly positive rational components. Scaling $S$ by $Y$ preserves the feasible sequences of firings.*

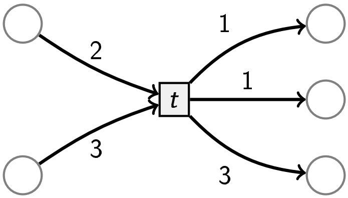
# Balancing - 1-conservativeness

## Definition

A transition $t$ is 1-conservative if

$$\sum_{p \in {}^{\bullet}t} W(p,t) = \sum_{p \in t^{\bullet}} W(t,p)\,.$$

If all the transitions of a net are 1-conservative, the net is said to be 1-conservative.



# Balancing - 1-conservativeness

**Lemma**

1-*conservativeness implies conservativeness.*

# Balancing - 1-conservativeness

**Definition**

Let $S$ be a system. Balancing $S$ consists in scaling $S$ by a vector $Y$ of strictly positive rational numbers such that the resulting system is 1-conservative.

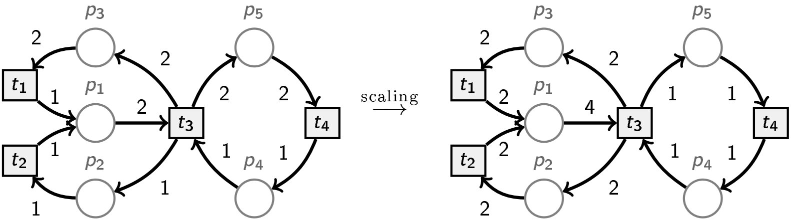



The system on the left is balanced by $(2, 2, 1, 1, \frac{1}{2})$, yielding the system on the right.

# Balancing - 1-conservativeness

## Definition

Let $S$ be a system. Balancing $S$ consists in scaling $S$ by a vector $Y$ of strictly positive rational numbers such that the resulting system is 1-conservative.




The system on the left is balanced by $(2, 2, 1, 1, \frac{1}{2})$, yielding the system on the right.

# Balancing - 1-conservativeness

**Lemma**

*A system is conservative if and only if it can be balanced.*

# Balancing - 1-conservativeness

**Theorem**

*Balancing preserves the feasible sequences.*

**Corollary**

*A conservative system is live if and only if one of its balancings is live.*

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# Useful tokens

**Definition**

A weighted Petri net is said to satisfy the useful tokens condition if every place $p$ is initially marked with a multiple of $gcd_p$.

**Theorem**

*The marking $M_0(p)$ of every place $p$ of a system $S = (N, M_0)$ can be replaced by*

$$\left\lfloor \frac{M_0(p)}{gcd_p} \right\rfloor \cdot gcd_p$$

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# Outline

4 Results
- Polynomial transformations preserving the sequences of firings
  - Scaling
  - Balancing
  - Useful tokens
- Well-behavedness of Join-Free systems
- Well-behavedness of Choice-Free systems
- Sufficient conditions are not necessary

# On the road to liveness ... of Join-Free systems

1. Under certain conditions, a balanced Join-Free system can benefit from the existence of at least one enabled place at any reachable marking
2. Such conditions lead to well-behavedness

# Enabled places in Join-Free systems

## Lemma

*Let $S = ((P, T, W), M_0)$ be a balanced strongly connected Join-Free system fulfilling the useful tokens condition and the inequality*

$$\sum_{p \in P} M_0(p) > \sum_{p \in P} (max_p - gcd_p) \,.$$

*Then for every marking $M$ in $[M_0\rangle$, there exists a place $p \in P$ which is enabled by $M$.*

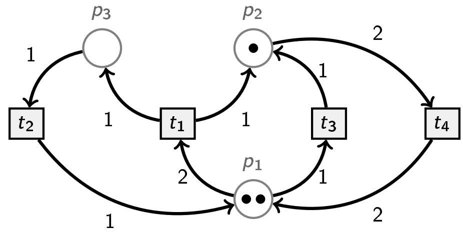
# A sufficient condition of well-behavedness
## for balanced well-formed Join-Free nets

**Theorem**

*Let $S = (N, M_0)$ be a balanced strongly connected Join-Free system satisfying the useful tokens condition. S is live if*

$$\sum_{p \in P} M_0(p) > \sum_{p \in P} (max_p - gcd_p) \,.$$

# A well-behaved balanced Join-Free system




The initial marking of this balanced Join-Free system fulfills the conditions of the theorem and is thus well-behaved.

$$I = \sum_p M_0(p) = 3$$

$$C = \sum_p (max_p - gcd_p) = 1 + 1 + 0 = 2$$

$I > C$ thus the system is well-behaved.

# Outline

4. Results
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# On the road to liveness ... of Choice-Free systems

1. An existing result characterizes the liveness of Choice-Free systems in terms of the liveness of their Fork-Attribution P-subnets.
(Teruel, Colom, Silva 97)

2. The previous sufficient condition of well-behavedness for Join-Free systems applies to FA systems

3. In so doing, we obtain another sufficient condition of well-behavedness for Choice-Free systems

# Liveness and FA P-subnets

A source place is defined as a place with at least one output transition and without input transition.

**Theorem (Teruel, Colom, Silva 97)**

*Let $(N, M_0)$ be a Choice-Free system without source places. $(N, M_0)$ is live iff for every strongly connected FA P-subnet of $N$, noted $N'$, the system $(N', M_0[P'])$ is live.*

# Liveness of FA systems

The condition of liveness for Join-Free systems applies to FA systems :

**Lemma**

*Let $S = ((P, T, W), M_0)$ be a strongly connected and balanced FA system satisfying the useful tokens condition. $S$ is live if*

$$\sum_{p \in P} M_0(p) > \sum_{p \in P} (max_p - gcd_p) .$$

and is simplified as follows to be useful for Choice-Free systems :

**Lemma**

*Let $N = (P, T, W)$ be a strongly connected and conservative FA net and $M_0$ an initial marking such that*

$$\forall p \in P, \ M_0(p) = max_p$$

*then the FA system $(N, M_0)$ is live.*

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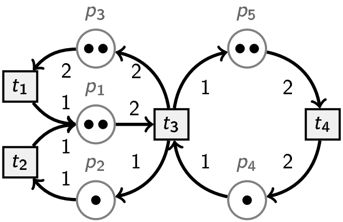
# A sufficient condition of well-behavedness

## for well-formed Choice-Free nets

### Theorem

*Let $S = ((P, T, W), M_0)$ be a well-formed Choice-Free system.*
*$S$ is well-behaved if*

$$\forall p \in P, \ \ M_0(p) = max_p \, .$$




Each place $p$ is initially marked with $max_p$ tokens : this Choice-Free system is well-behaved.

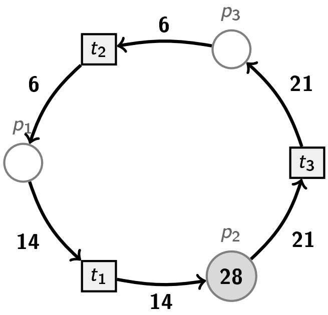
# Outline

4. Results
   - Polynomial transformations preserving the sequences of firings
     - Scaling
     - Balancing
     - Useful tokens
   - Well-behavedness of Join-Free systems
   - Well-behavedness of Choice-Free systems
   - **Sufficient conditions are not necessary**

# Sufficient conditions are not necessary

Both previous sufficient conditions of liveness for Join-Free and Choice-Free systems are not necessary.

$$\sum_p (max_p - gcd_p) = (14 - 2) + (21 - 7) + (6 - 3) = 29\,.$$




This circuit is a live Join-Free and Choice-Free system but does not fulfill their sufficient condition.

# Outline

1. Designing embedded systems
2. Petri nets : definitions and properties
3. Relevance of the study
4. Results
5. Conclusion and outlook

# Conclusion and outlook

## Key points

- Weights are useful : they provide more flexibility
- Well-formedness and well-behavedness are two relevant properties of well-designed systems
- Ensuring the well-behavedness of Choice-Free and Join-Free systems requires no longer exponential time, but polynomial time
- Simple polynomial time Petri nets transformations may be reused in other contexts

## Outlook

Finding throughput bounds of temporized Choice-Free and Join-Free systems (work in progress)

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