

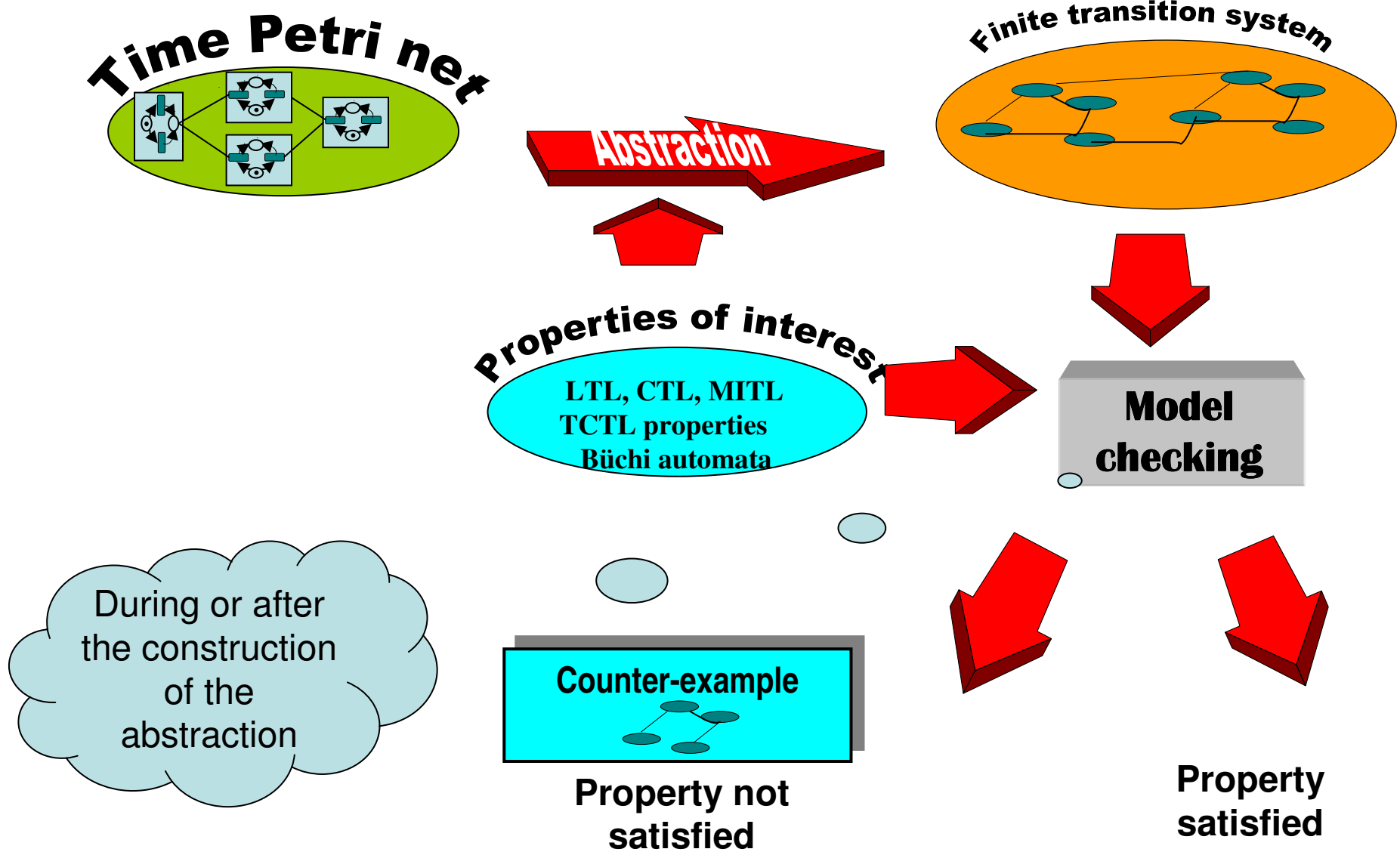
# Model checking timed linear properties of time Petri nets using the state class method

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# Model checking of time Petri nets based on the state class method

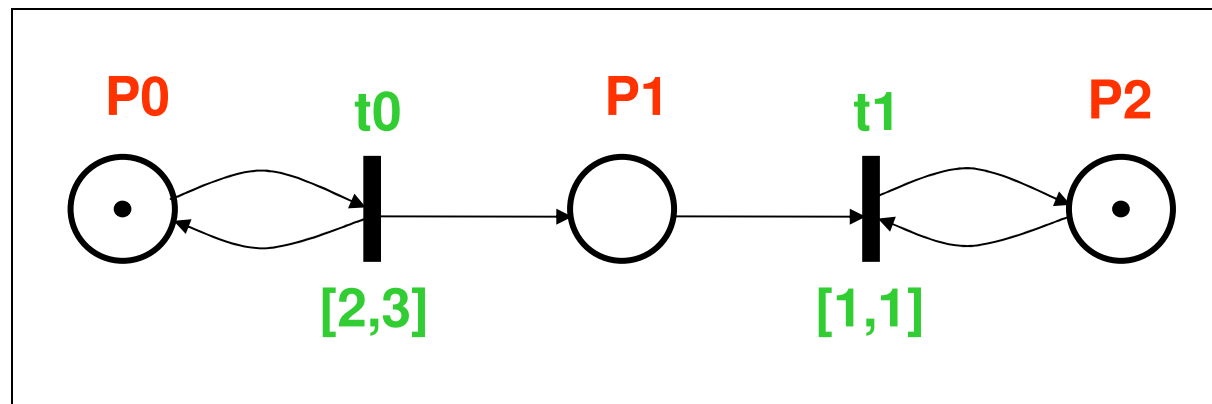


# Plan

- **Time Petri Nets (TPN model)**
- **TPN state space abstractions**
- **The state class graph method (SCG)**
- **Contracting the state class graph (CSCG)**
- **Model checking using an interval timed extension of Büchi Automata (ITBA)**
- **Conclusion**

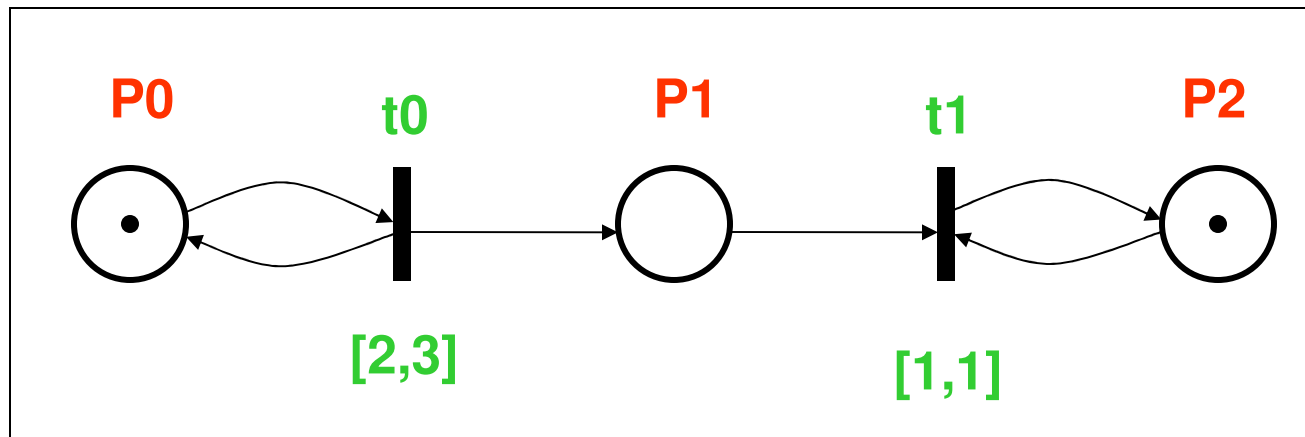
# Time Petri Nets (Merlin & Farber 1976)

- Extension of Petri Nets by associating **timing constraints with transitions in the form of intervals** (static firing intervals)



→ a good compromise between **modeling Power** and **verification possibilities**

# TPN State : Interval state vs. Clock state



Two state definitions:

**Interval State (M,I)**

$P_0=1, P_1=0, P_2=1$

$I(t_0) = [2, 3]$

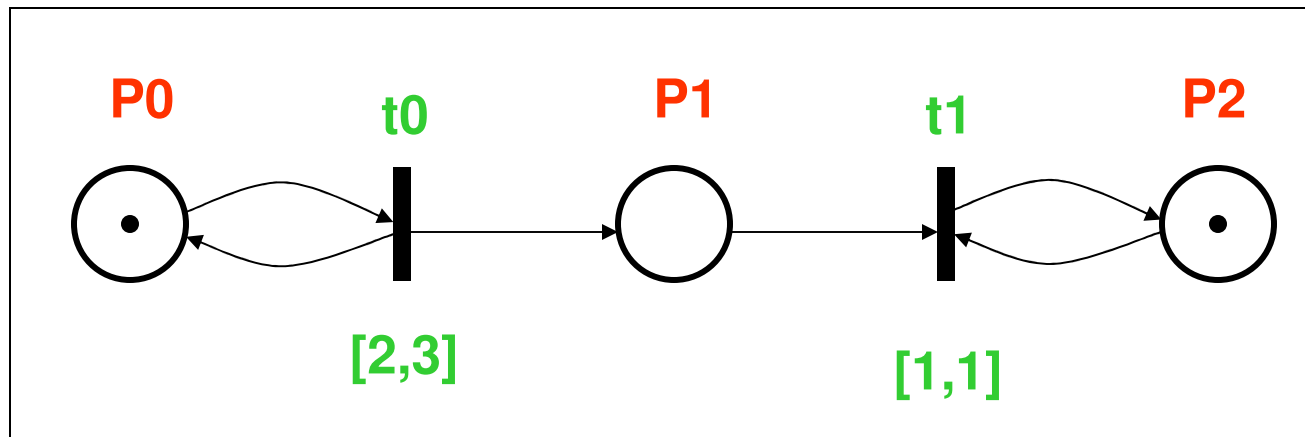
**Clock State (M,v)**

$P_0=1, P_1=0, P_2=1$

$v(t_0) = 0$

# State evolution

TPN state evolves either by time progression or by firing transitions

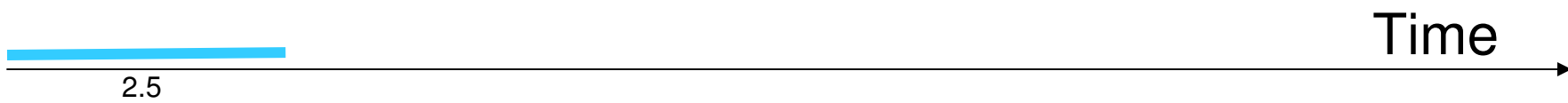


bounds of intervals decrease with time

**Interval State (M,I)**  
**P0=1, P1=0, P2=1**  
 $I(t_0) = [0, 0.5]$

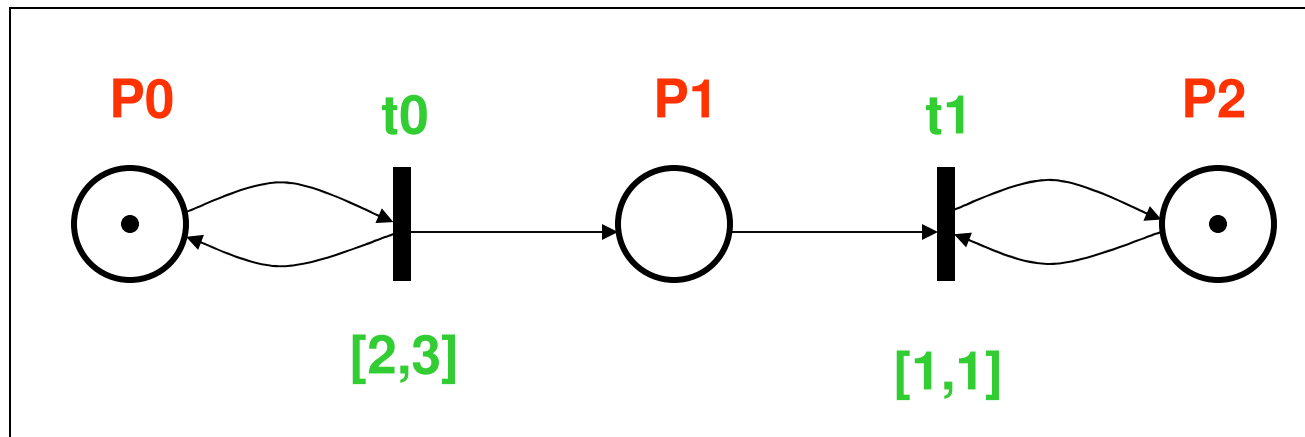
**Clock State (M,v)**  
**P0=1, P1=0, P2=1**  
 $v(t_0) = 2.5$

Clocks increase with time



# State evolution

TPN state evolves either by time progression or by firing transitions

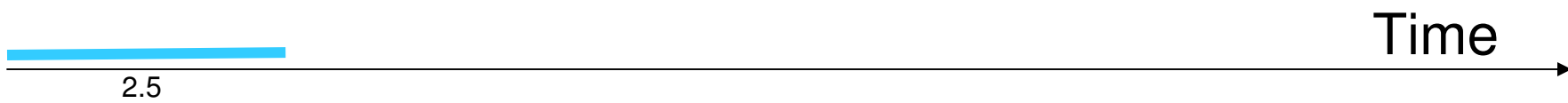


lower bound reaches 0  
 → t0 is firable

**Interval State (M,I)**  
 P0=1, P1=0, P2=1  
 $I(t_0) = [0, 0.5]$

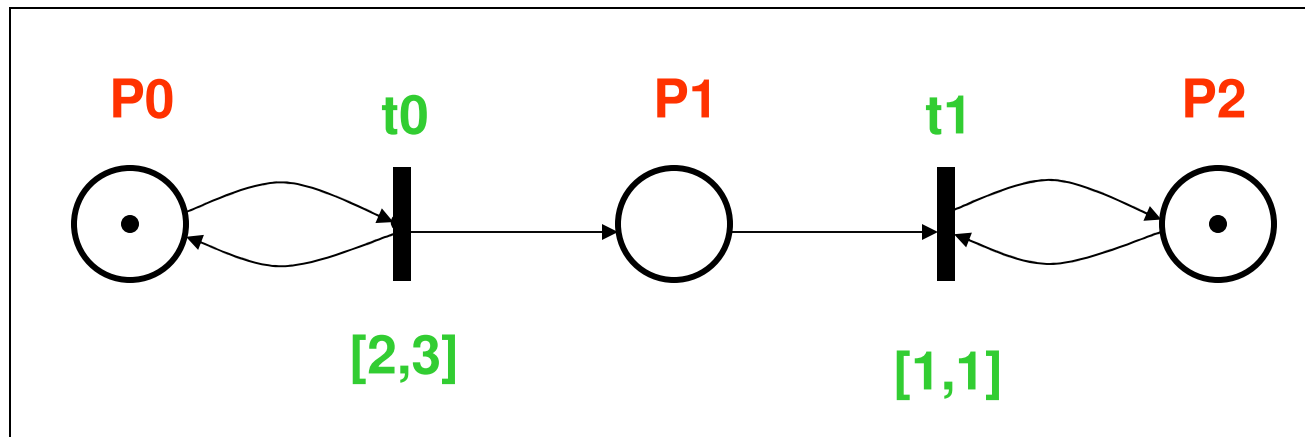
**Clock State (M,v)**  
 P0=1, P1=0, P2=1  
 $v(t_0) = 2.5$

Clock reaches the interval of t0  
 → t0 is firable



# State evolution

TPN state evolves either by time progression or by firing transitions



Firing may enable / disable transitions

**Interval State (M,I)**  
**P0=1, P1=1, P2=1**  
 $I(t0) = [2, 3]$   
 $I(t1) = [1, 1]$

**Clock State (M,v)**  
**P0=1, P1=1, P2=1**  
 $v(t0) = 0$   
 $v(t1) = 0$

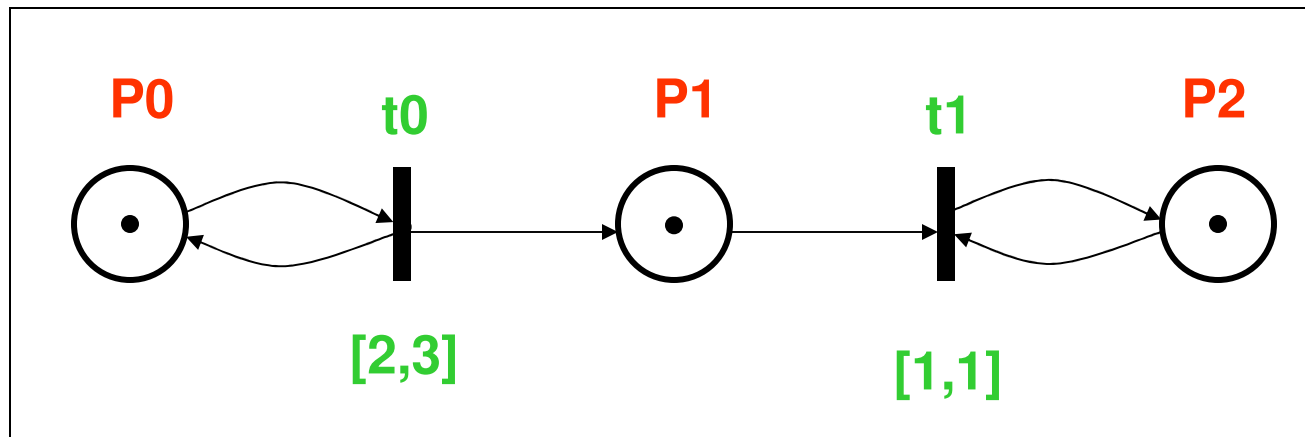
Firing may enable / disable transitions





# State evolution

TPN state evolves either by time progression or by firing transitions



Upper bound reaches 0  
 → t1 must be fired

**Interval State (M,I)**  
 P0=1, P1=1, P2=1  
 $I(t0) = [1, 2]$   
 $I(t1) = [0, 0]$

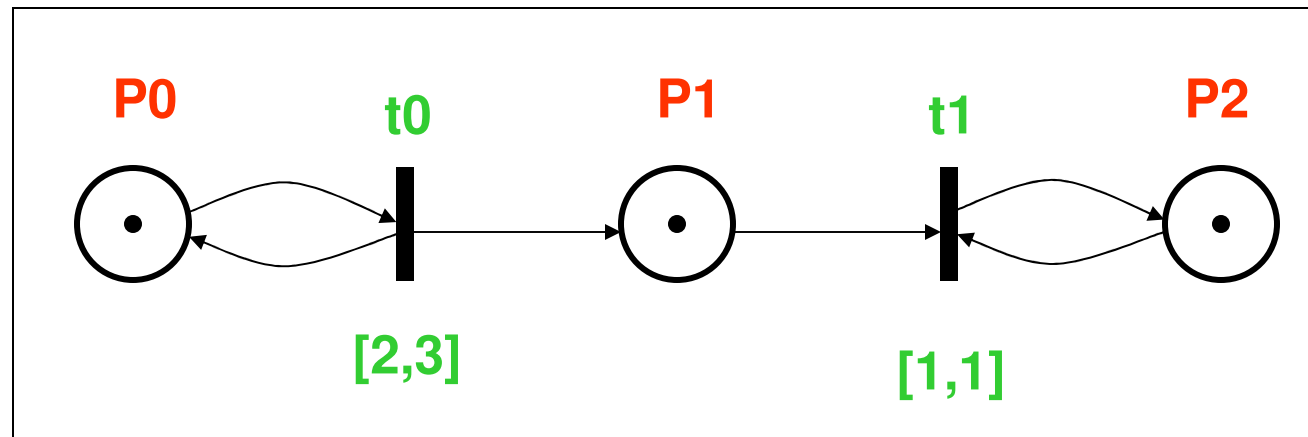
**Clock State (M,v)**  
 P0=1, P1=1, P2=1  
 $v(t0) = 1$   
 $v(t1) = 1$

Clock reaches the upper bound  
 → t1 must be fired



# State evolution

TPN state evolves either by time progression or by firing transitions



**Interval State (M,I)**

$P_0=1, P_1=0, P_2=1$

$I(t_0) = [1, 2]$

**Clock State (M,v)**

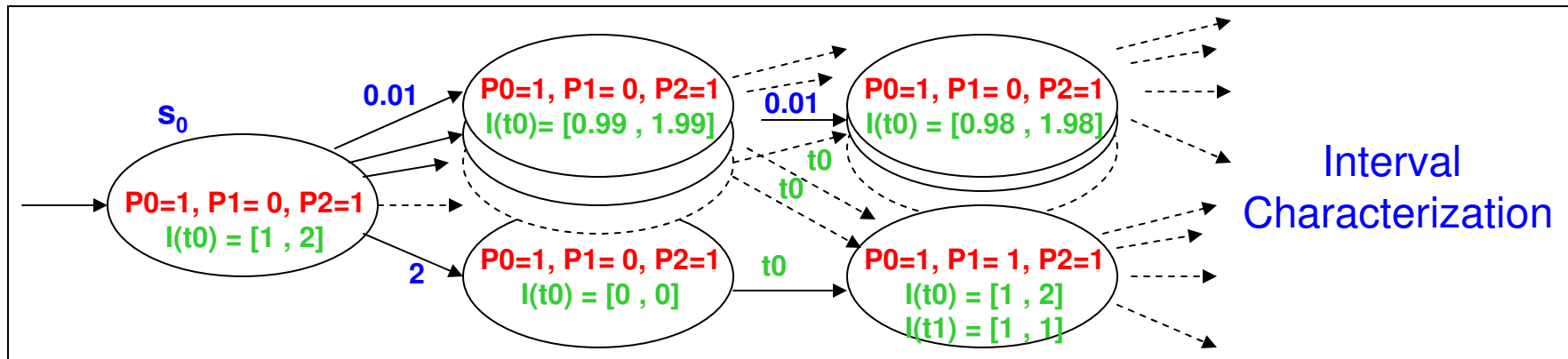
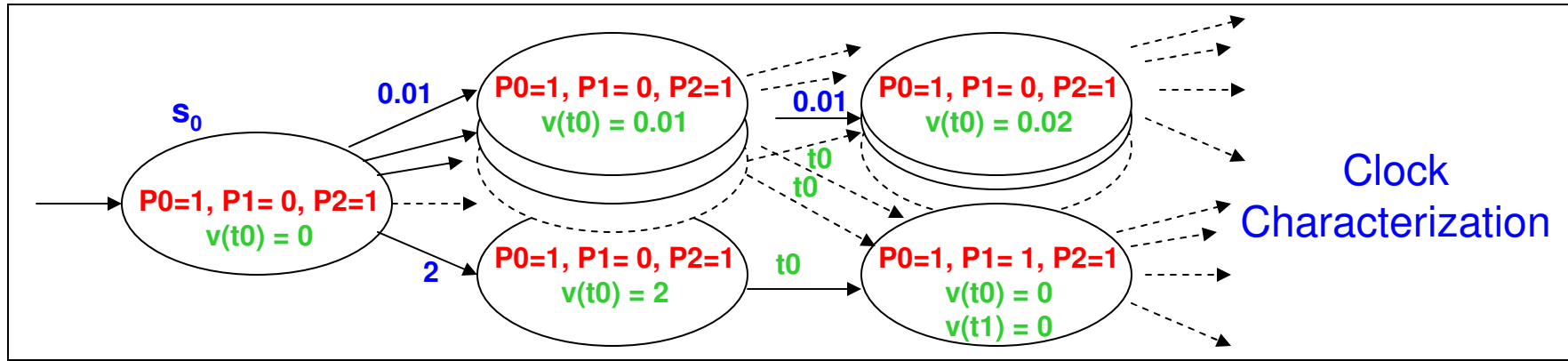
$P_0=1, P_1=0, P_2=1$

$v(t_0) = 1$



# TPN state space

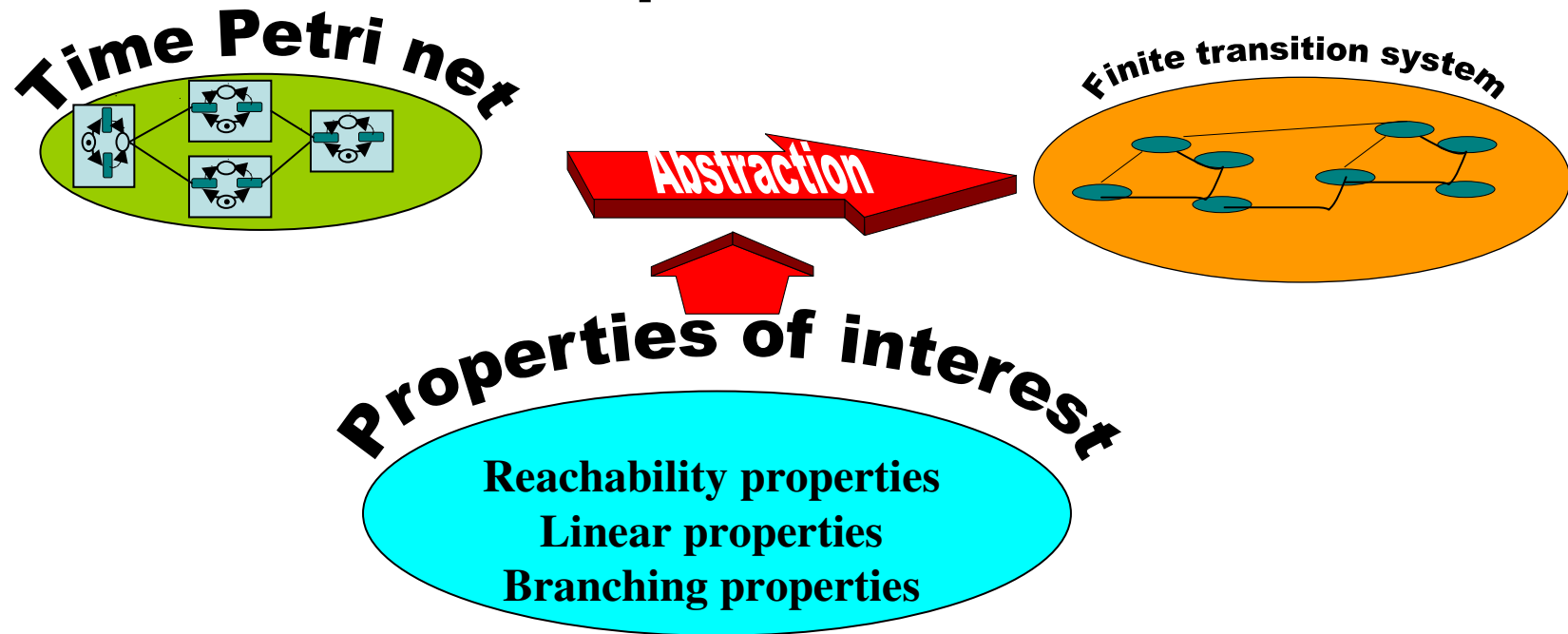
Transition system  $(S, \rightarrow, s_0)$



**Infinite state space with infinite branching**

**How to abstract the TPN state space ?**

# TPN state space **abstractions**



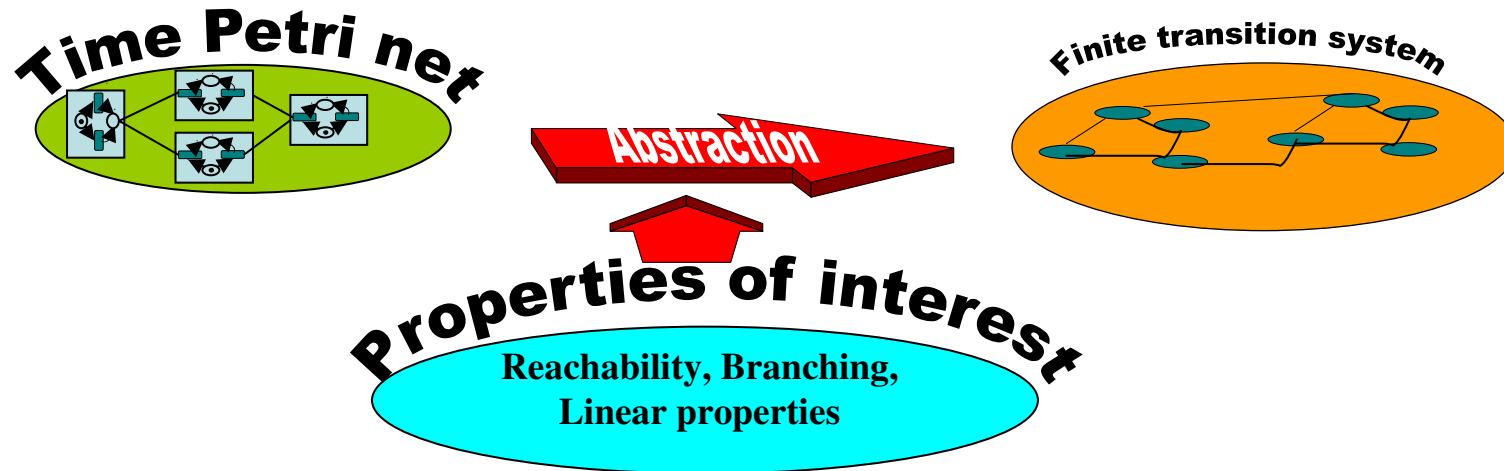
Abstraction  $\rightarrow$  Abstract irrelevant information while preserving properties of interest:

- Reachability: markings or states of the model.
- **Linear properties: firing sequences of the model.**
- Branching properties: execution trees of the model.

Challenge:

- **More coarser finite abstraction preserving properties of interest.**
- **Computed with minor resources (time and space).**

# TPN state space abstractions



TPN state space abstractions in the literature preserving linear properties:

- SCG [Berthomieu & Diaz 1991],
- GRG [Yoneda & Ryuba 1998],
- SSCG [Berthomieu & Vernadat 2003],
- ZBG [Gardey & Roux 2003],
- CSZG [Hadjidj & Boucheneb 2005].
- ASCGs

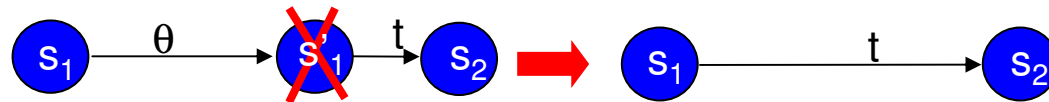
→ may differ in:

- Characterization of states ( **interval state abstractions, clock state abstractions** ),
- Agglomeration criteria of states
- Preserved properties.

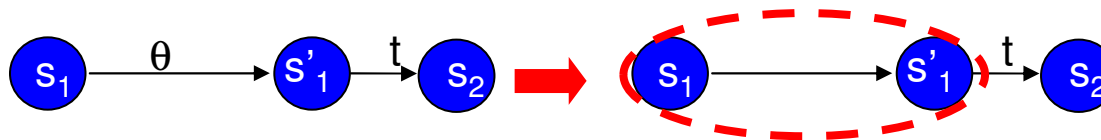
# TPN state space abstractions

Three levels of abstraction:

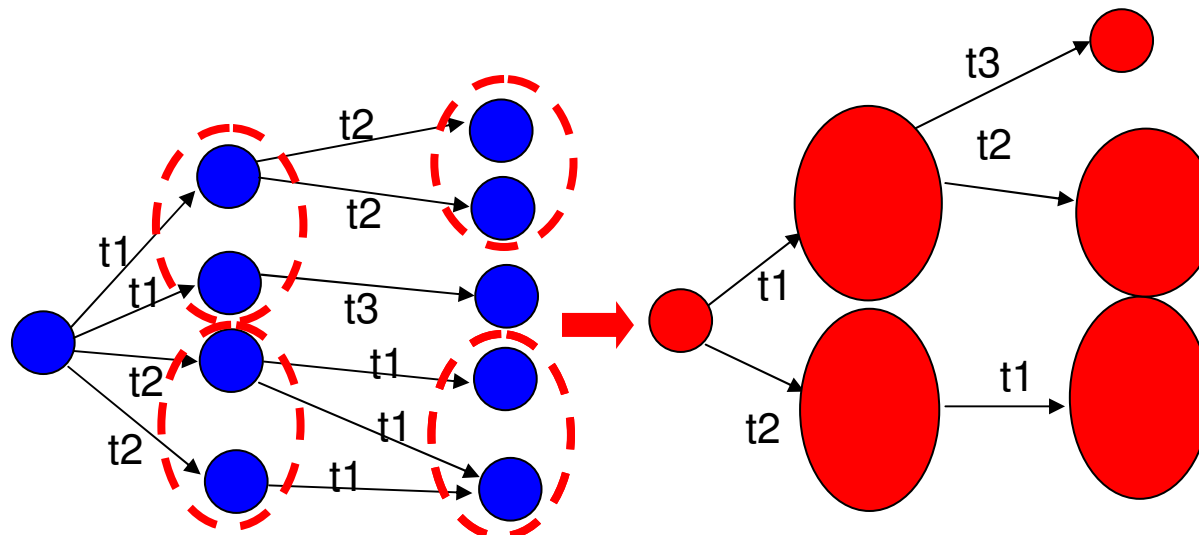
## 1. Time abstraction



OR



## 2. States reachable by the same firing sequence independently of their firing times are agglomerated in the same node.

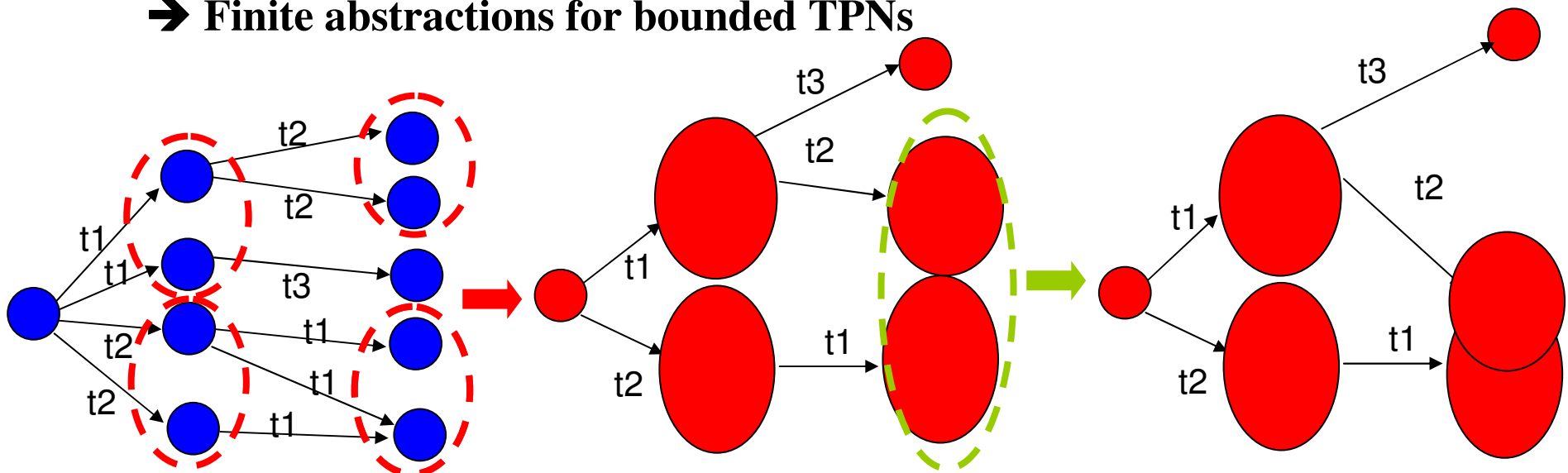


# TPN state space **abstractions**

3. Agglomerated states are then considered modulo some relation of equivalence:

- **Firing domain of the SCG** [Berthomieu & Diaz 1991],
- **k-approximation of the ZBG** [Gardey & Roux 2003],
- **Approximation of the SSCG** [Berthomieu & Vernadat 2003],
- **k-normalization of the CSZG** [Hadjidj & Boucheneb 2005],
- **Relation of equivalence of the GRG** [Yoneda & Ryuba 1998].

➔ **Finite abstractions for bounded TPNs**





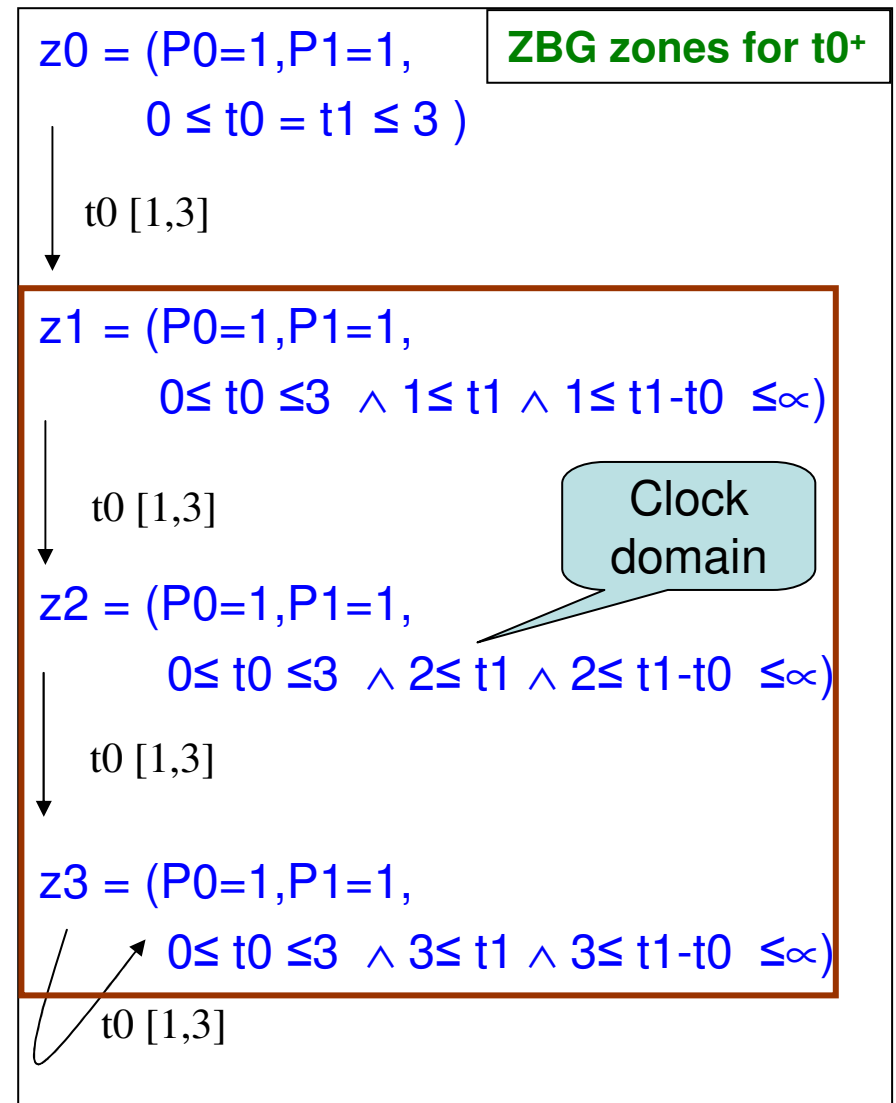
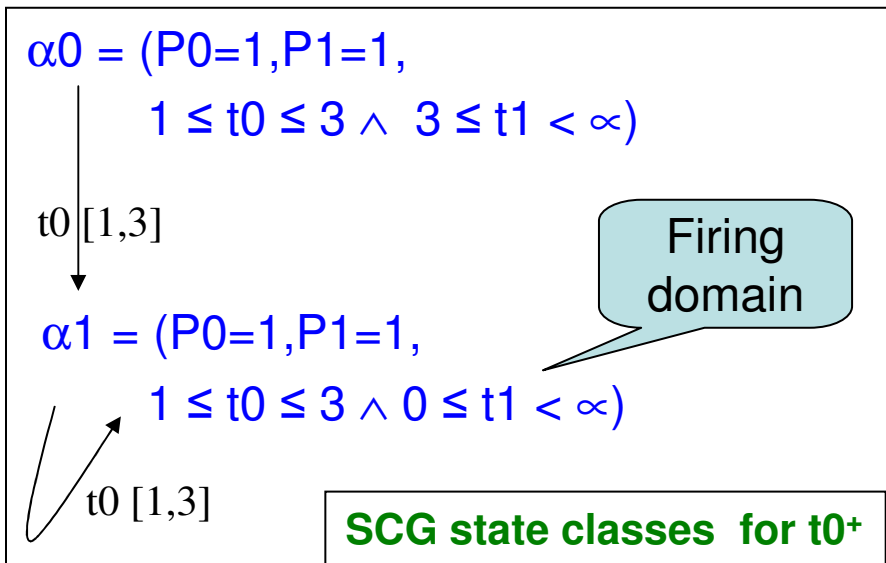
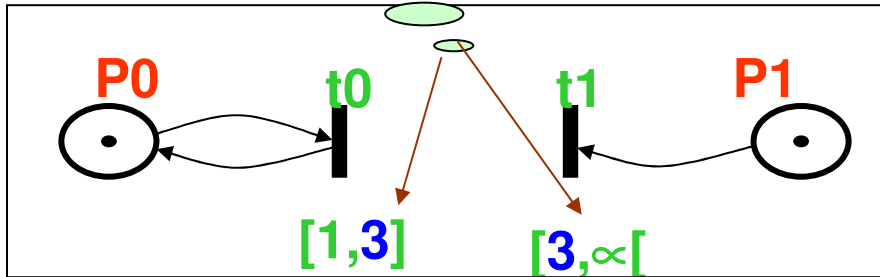
# TPN state space **abstractions**

The SCG is the more interesting abstraction. Why?

- The SCG is an interval state abstraction; all others are clock state abstractions.
  - Abstractions **based on clocks** do not enjoy naturally the **finiteness property** for **bounded TPNs** with **unbounded intervals**. The **finiteness is enforced** using:
    - k-approximation of the ZBG [ Gardey & Roux 2003],
    - Approximation of the SSCG [Berthomieu & Vernadat 2003],
    - k-normalization of the CSZG [Hadjidj & Boucheneb 2005].
- may involve some **overhead computation**.
- The **interval characterization** of states has a **more abstracting power** than the **clock characterization**, and allows to construct **more compact abstractions**.

# TPN state space **abstractions**

The number of zones depends on the value of these bounds (3+1)



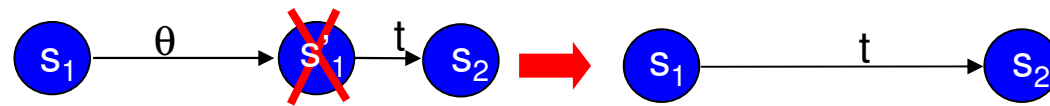
- Many ZBG zones may map to a single SCG state class.

# **The state class graph method**

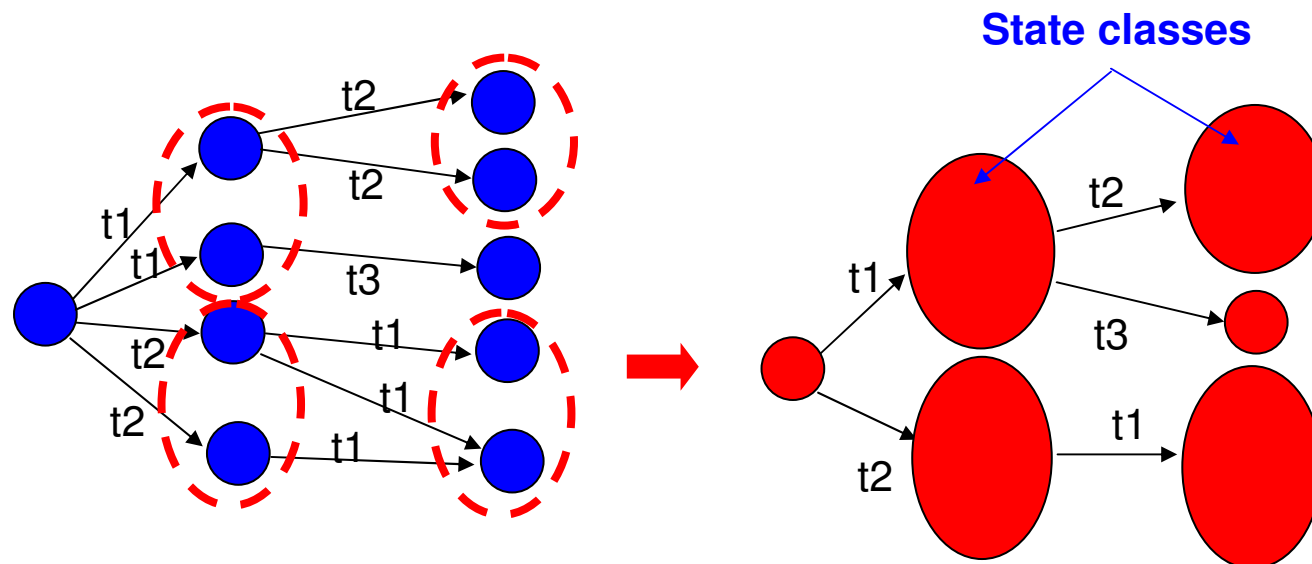
# The state class graph method

1. State characterization: interval state  $(M,I)$

2. Time abstraction:

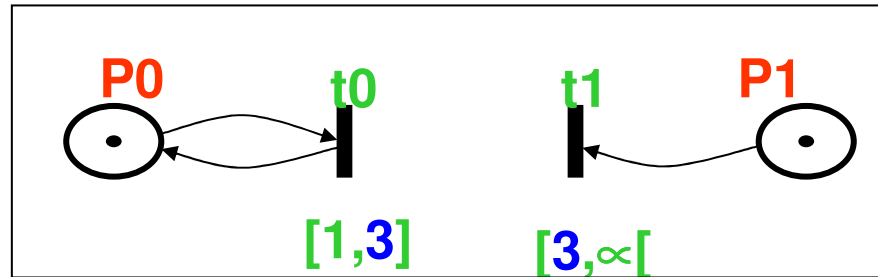


3. States reachable by the same firing sequence are agglomerated and considered modulo some relation of equivalence  $\rightarrow$  **firing domain**



$\rightarrow$  A state class = (marking, set of constraints which characterizes its states)

# The state class graph method



- The initial state class:  $\alpha_0 = (P_0=1, P_1=1, 1 \leq t_0 \leq 3 \wedge 3 \leq t_1 < \infty)$
- $\text{succ}(\alpha_0, t_0) \neq \emptyset$  iff  $t_0$  is enabled and the following formula is consistent  
 $1 \leq t_0 \leq 3 \wedge 3 \leq t_1 < \infty \wedge t_0 \leq t_1$
- The firing of  $t_0$  leads to the state class  $\alpha_1 = (P_0=1, P_1=1, F_1)$  where  $F_1$  is computed in four steps:
  1.  $F_1 \leftarrow 1 \leq t_0 \leq 3 \wedge 3 \leq t_1 < \infty \wedge t_0 \leq t_1$  t1 = new t1+t0
  2. Replace  $t_1$  with  $t_1+t_0$ :  $1 \leq t_0 \leq 3 \wedge 3 \leq t_1+t_0 < \infty \wedge t_0 \leq t_1+t_0$
  3. Eliminate by substitution  $t_0$  and conflicting transitions:  $0 \leq t_1 < \infty$
  4. Add constraints of newly enabled transitions:  $0 \leq t_1 < \infty \wedge 1 \leq t_0 \leq 3$

# The state class graph method

- $F$  is a conjunction of atomic constraints:  $t_i - t_j < c$ ,  $t_i < c$ ,  $-t_j < c$ , where  $t_i, t_j \in T$ ,  $c \in Q \cup \{\alpha, -\alpha\}$ ,  $< \in \{=, \leq, \geq\}$

- Canonical form of  $F$ : 
$$\bigwedge_{x, x' \in \text{En}(M) \cup \{o\}} x - x' \leq \text{Sup}_F(x - x')$$

where  $o$  represents the value zero and

$\text{Sup}_F(x - x')$  is the least upper bound of  $x - x'$  in the domain of  $F$

- It can be represented by a Difference Bound Matrix (DBM)  $D$ :

$$\forall x, x' \in \text{En}(M) \cup \{o\}, \quad D_{xx'} = \text{Sup}_F(x - x')$$

- $F$  and all formulae equivalent to  $F$  have **the same canonical form**.
- Canonical form of  $F$  may be computed using:
  - The shortest path Floyd-Warshall's algorithm in  $O(n^3)$ ,  $n$  being the number of variables in  $F$ , or
  - The algorithm, proposed in [Boucheneb & Mullins 2003], in  $O(n^2)$ .

# The state class graph method

Algorithm proposed in [Boucheneb & Mullins 2003]

Let  $\alpha=(M,D)$  be a state class in canonical form and  $t_f$  a transition.

- $t_f$  is firable from  $(M, D)$  (i.e.:  $Succ(\alpha, t_f) \neq \emptyset$ ) iff  $t_f \in En(M)$  and  $Min_{t \in En(M)} D_{t t_f} = 0$ .

Time complexity  
 $O(n)$

- If  $t_f$  is firable from  $(M, D)$ , its firing leads to the state class  $Succ(\alpha, t_f) = (M', D')$  computed as follows:  
 $\forall p \in P, M'(p) = M(p) - Pre(p, t_f) + Post(p, t_f);$   
 $\forall t \in En(M'),$

$$D'_{t o} = \begin{cases} tmax(t) & \text{if } t \in New(M', t_f); \\ D_{t t_f} & \text{otherwise} \end{cases}$$

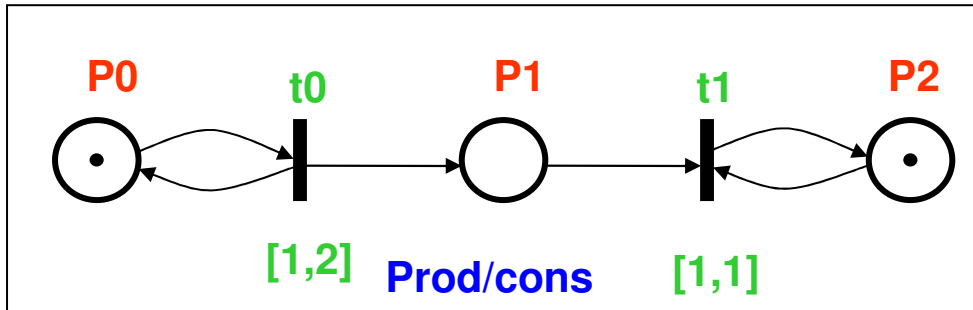
$$D'_{o t} = \begin{cases} -tmin(t) & \text{if } t \in New(M', t_f); \\ Min_{u \in En(M)} D_{u t} & \text{otherwise} \end{cases}$$

$$\forall (t, t') \in (En(M'))^2,$$

$$D'_{t t'} = \begin{cases} 0 & \text{if } t \text{ is } t' \\ D'_{t o} + D'_{o t'} & \text{if } t \text{ or } t' \in New(M', t_f); \\ Min(D_{t t'}, D'_{t o} + D'_{o t'}) & \text{otherwise} \end{cases}$$

Time complexity  
 $O(n^2)$

# An example



Reachable markings:

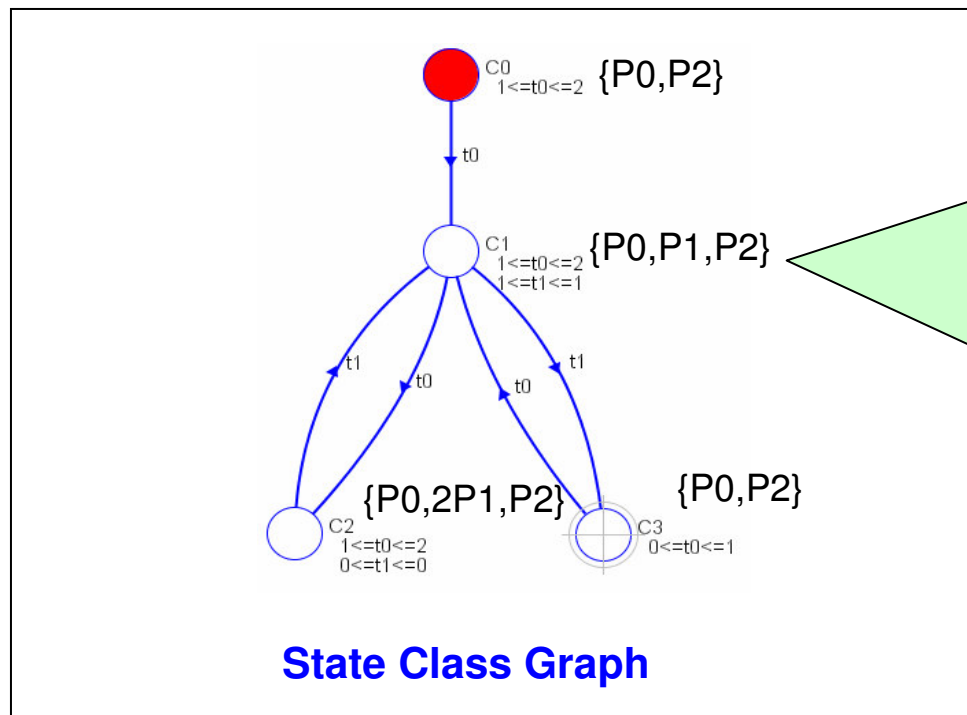
1.  $P_0 = 1, P_2 = 1$
2.  $P_0 = 1, P_1 = 1, P_2 = 1$
3.  $P_0 = 1, P_1 = 2, P_2 = 1$

Firing sequences:  $t_0; \{ t_0; t_1, t_1; t_0 \}^\infty$

Path / cycle times of firing sequences [Boucheneb & Mullins 2003]:

From  $C_0$ :  $t_0; t_1 \rightarrow [2, 3]$

From  $C_1$ :  $t_0; t_1 \rightarrow [1, 2]$



State Class Graph

→ SCG is finite iff the TPN is bounded.

→ SCG preserves markings and traces of the TPN → Linear properties

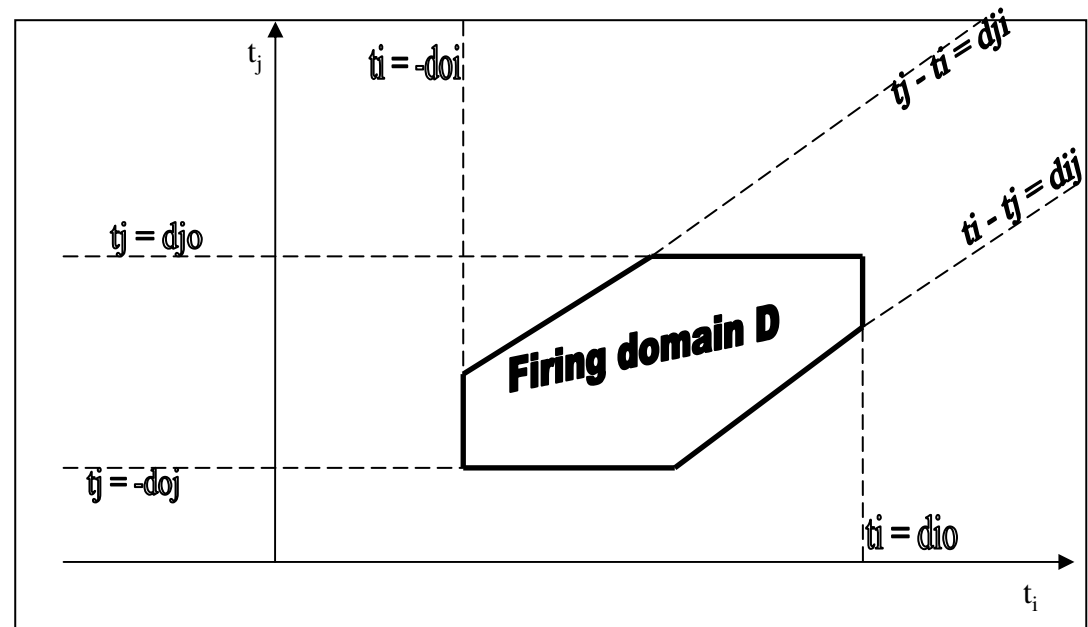


# **Contracting the state class graph**

# Contracting the state class graph

- Define a *relation of equivalence* over *state classes* such that *equivalent classes* have the *same firing sequences*.
  - Construct a *quotient graph* w.r.t. the *relation of equivalence*.
- *Over-approximation of state classes* [Boucheneb & Rakkay; ACSD 2007]:

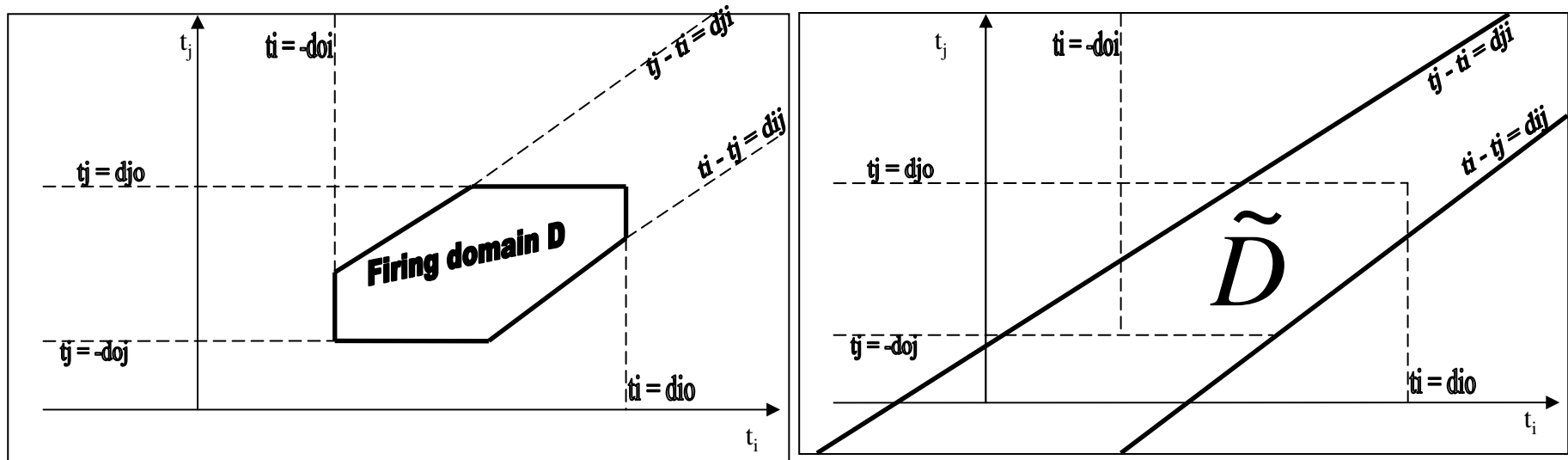
<p>D:</p> <ul style="list-style-type: none"> <li>- <math>doi \leq t_i \leq dio</math></li> <li>- <math>doj \leq t_j \leq djo</math></li> <li>- <math>dji \leq t_i - t_j \leq dij</math></li> </ul>
--



# Over-approximation of state classes

- *The idea of this operation comes from:*
    - 1- The **line and column 0** of each DBM  $D$  are not necessary to compute **successor state classes** of  $(M,D)$  (they can be eliminated)
      - a gain in space
    - 2- The **firing of any transition  $t_f$**  from a state class  $\alpha=(M,D)$ , will **disable all transitions conflicting with  $t_f$** 
      - There is no need to compute, for the *successor of  $\alpha$  by  $t_f$*  distances between  $t_f$  and its conflicting transitions,
      - but we need to know whether  $t_f$  is *firable or not before all enabled transitions (including those in conflict with  $t_f$ )*.
- ⇒ *State classes with lesser constraints*

# Over-approximation of state classes



$$\tilde{d}_{ij} = \begin{cases} \infty & \text{if } (i = 0 \text{ and } j \neq 0) \text{ or } (j = 0 \text{ and } i \neq 0) \\ 0 & \text{if } (i, j \neq 0) \text{ and } d_{ij} \geq 0 \\ & \text{and } (t_i \text{ and } t_j \text{ in conflict}) \\ d_{ij} & \text{otherwise} \end{cases}$$

Eliminate simple constraints

CSCG is the quotient graph of SCG w.r.t  $\approx$ :

$$\forall \alpha_1, \alpha_2 \in C, \alpha_1 \approx \alpha_2 \text{ iff } \tilde{\alpha}_1 = \tilde{\alpha}_2$$

**Lemma:**  $\approx$  is a bisimulation over the SCG

# Contracting the state class graph

**Proposition 2** Let  $\alpha = (M, D)$  be a state class,  $\tilde{\alpha} = (M, \tilde{D})$  its equivalence class and  $t_f$  a transition.

$Succ(\alpha, t_f) \neq \emptyset$  iff  $Min_{u \in En(M)} \tilde{D}_{u t_f} = 0$ .

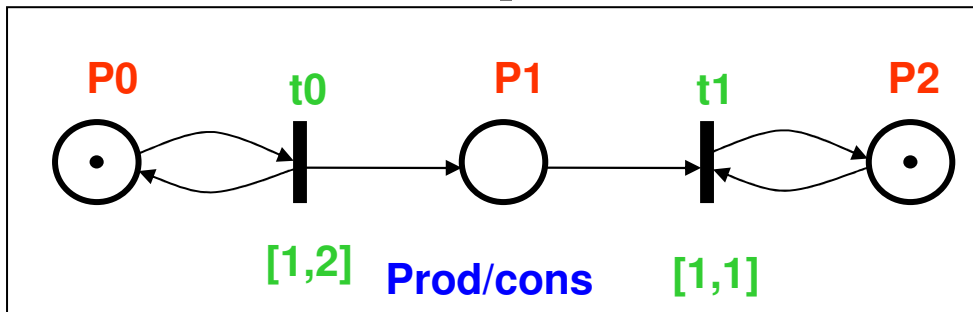
If  $Succ(\alpha, t_f) \neq \emptyset$  and  $Succ(\alpha, t_f) = (M', D')$  then the equivalence class  $(M', \tilde{D}')$  of  $(M', D')$  can be computed using  $\tilde{D}$  as follows:

$$\forall (t, t') \in (En(M'))^2 - conf(M'), \quad \tilde{D}'_{t t'} = \begin{cases} 0 & \text{if } t \text{ is } t' \\ tmax(t) - tmin(t') & \text{if } t, t' \in New(M', t_f); \\ tmax(t) + Min_{u \in En(M)} \tilde{D}_{u t'} & \text{if } t \in New(M', t_f); \\ \tilde{D}_{t t_f} - tmin(t') & \text{if } t' \in New(M', t_f); \\ Min(\tilde{D}_{t t'}, \tilde{D}_{t t_f} + Min_{u \in En(M)} \tilde{D}_{u t'}) & \text{otherwise} \end{cases}$$

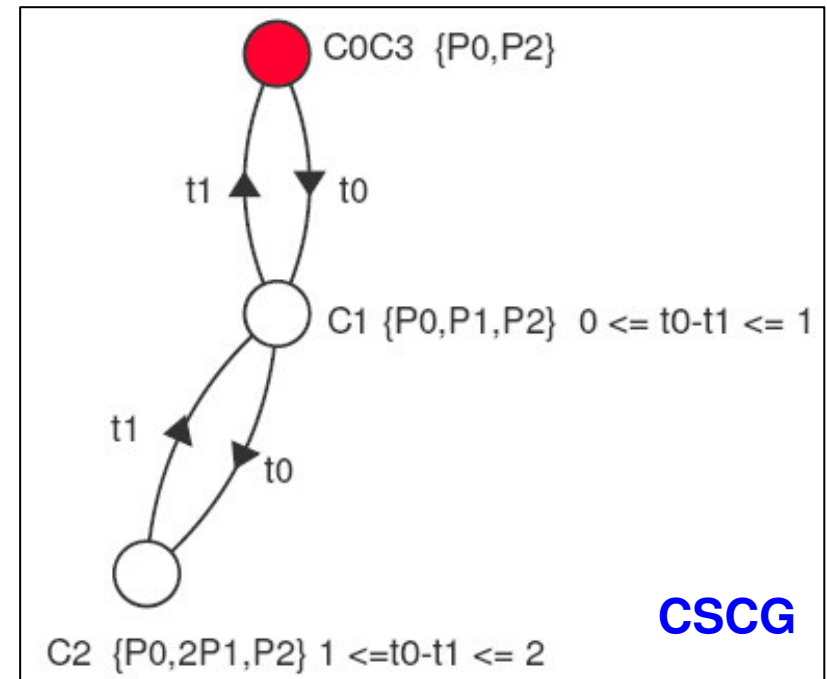
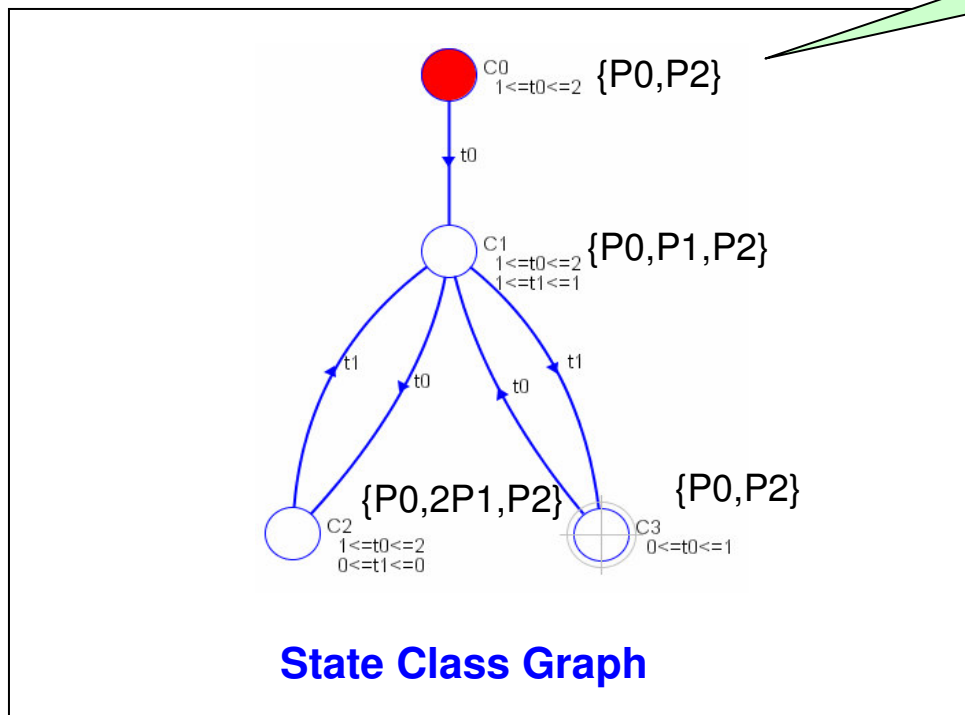
$$\forall (t, t') \in conf(M'), \quad \tilde{D}'_{t t'} = \begin{cases} 0 & \text{if } t \text{ is } t' \\ Min(0, tmax(t) - tmin(t')) & \text{if } t, t' \in New(M', t_f); \\ Min(0, tmax(t) + Min_{u \in En(M)} \tilde{D}_{u t'}) & \text{if } t \in New(M', t_f); \\ Min(0, \tilde{D}_{t t_f} - tmin(t')) & \text{if } t' \in New(M', t_f); \\ Min(0, \tilde{D}_{t t'}, \tilde{D}_{t t_f} + Min_{u \in En(M)} \tilde{D}_{u t'}) & \text{otherwise} \end{cases}$$

Classes of equivalence can be computed in  $O(n^2)$  directly from equivalence classes of their predecessors

# An example (no conflicting transitions)



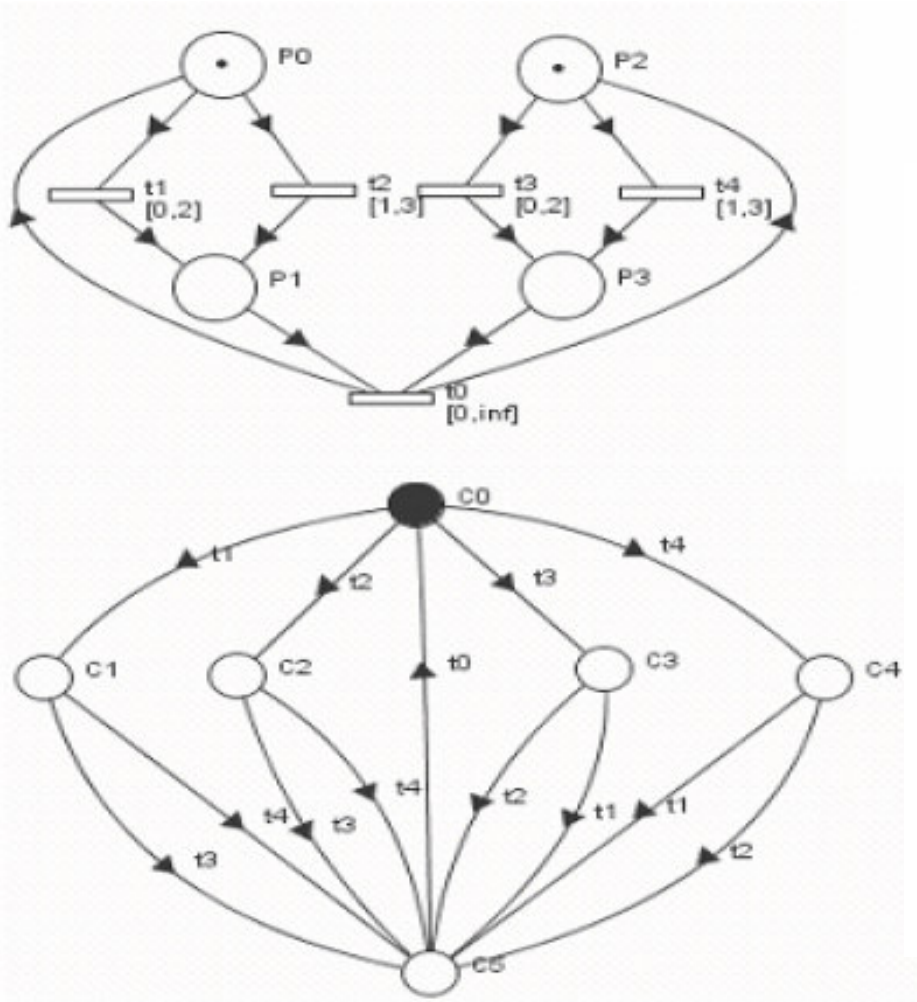
$C_0 \approx C_3$   
 $-\infty \leq t_0 \leq \infty$



- CSCG is smaller than CSG.
- CSCG preserves markings and traces of the TPN → Linear properties

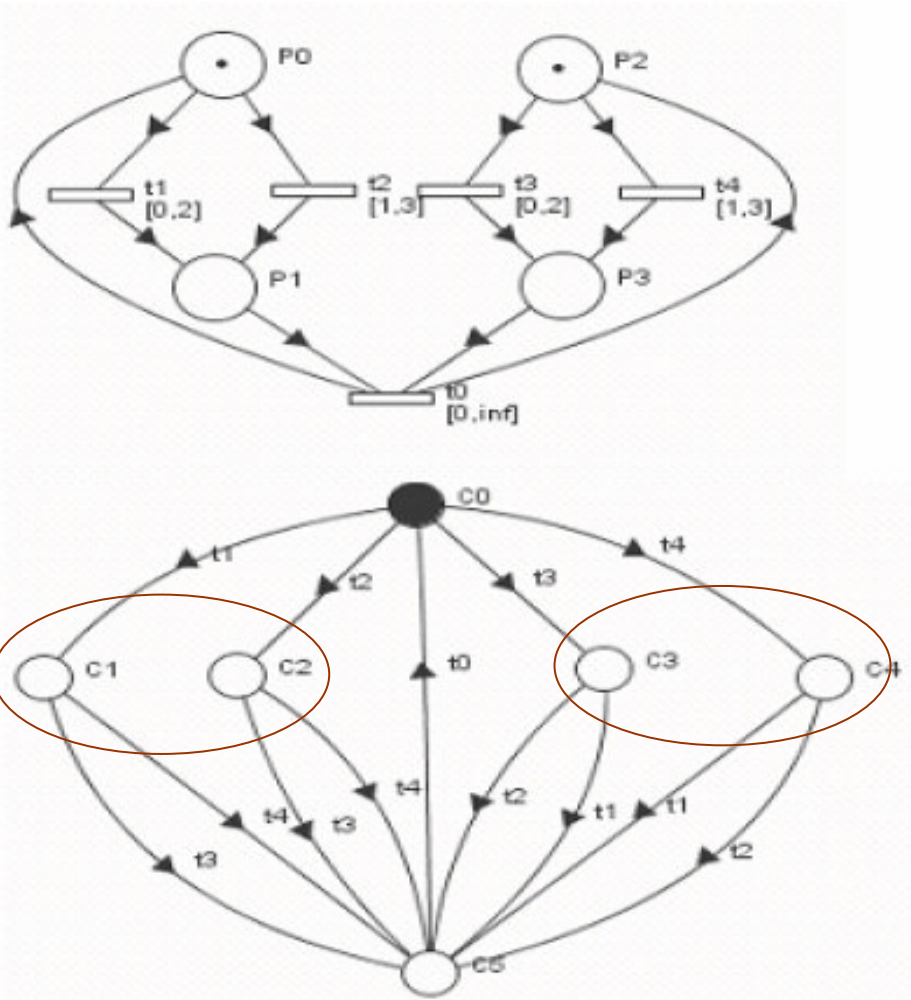
# Another example with conflicting transitions

Eliminating simple constraints



{	$C0 : P0 + P2$	<del><math>0 \leq t1 \leq 2</math></del>	<del><math>1 \leq t2 \leq 3</math></del>
		<del><math>0 \leq t3 \leq 2</math></del>	<del><math>1 \leq t4 \leq 3</math></del>
		$-3 \leq t1 - t2 \leq 1$	$-2 \leq t1 - t3 \leq 2$
		$-3 \leq t1 - t4 \leq 1$	$-1 \leq t2 - t3 \leq 3$
		$-2 \leq t2 - t4 \leq 2$	$-3 \leq t3 - t4 \leq 1$
{	$C1 : P1 + P2$	<del><math>0 \leq t3 \leq 2</math></del>	<del><math>0 \leq t4 \leq 3</math></del>
		$-3 \leq t3 - t4 \leq 1$	
{	$C2 : P1 + P2$	<del><math>0 \leq t3 \leq 1</math></del>	<del><math>0 \leq t4 \leq 2</math></del>
		$-2 \leq t3 - t4 \leq 1$	
{	$C3 : P0 + P3$	<del><math>0 \leq t1 \leq 2</math></del>	<del><math>0 \leq t2 \leq 3</math></del>
		$-3 \leq t1 - t2 \leq 1$	
{	$C4 : P0 + P3$	<del><math>0 \leq t1 \leq 1</math></del>	<del><math>0 \leq t2 \leq 2</math></del>
		$-2 \leq t1 - t2 \leq 1$	
{	$C5 : P1 + P3$	<del><math>0 \leq t0 \leq \infty</math></del>	

# Another example with conflicting transitions



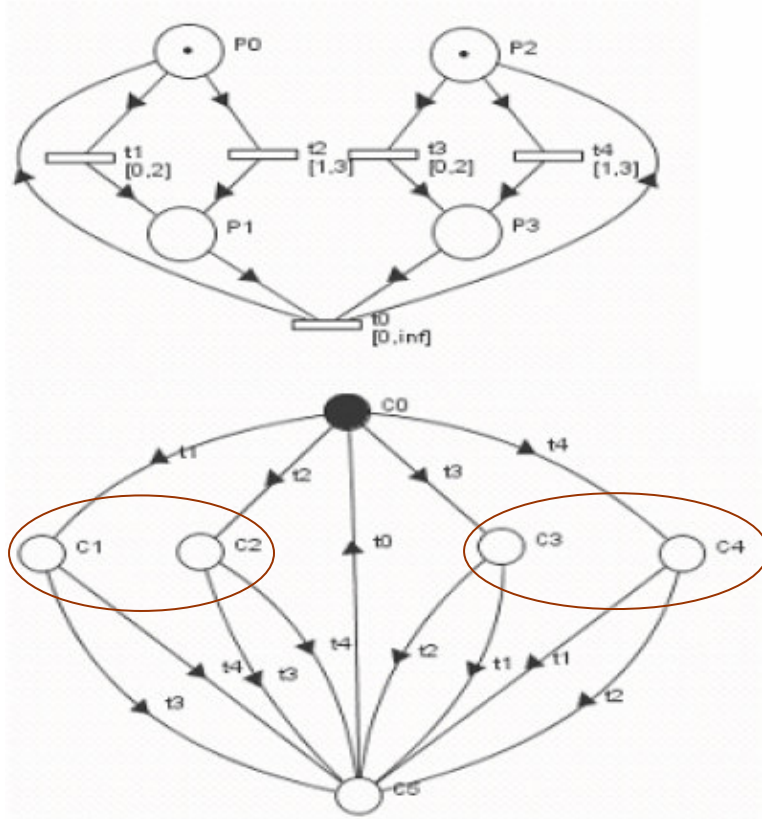
t3, t4 are in conflict

$C_0 : P_0 + P_2$	<del><math>0 \leq t_1 \leq 2</math></del>	<del><math>1 \leq t_2 \leq 3</math></del>
	<del><math>0 \leq t_3 \leq 2</math></del>	<del><math>1 \leq t_4 \leq 3</math></del>
	<del><math>-3 \leq t_1 - t_2 \leq 1</math></del>	<del><math>-2 \leq t_1 - t_3 \leq 2</math></del>
	<del><math>-3 \leq t_1 - t_4 \leq 1</math></del>	<del><math>-1 \leq t_2 - t_3 \leq 3</math></del>
	<del><math>-2 \leq t_2 - t_4 \leq 2</math></del>	<del><math>-3 \leq t_3 - t_4 \leq 1</math></del>
$C_1 : P_1 + P_2$	<del><math>0 \leq t_3 \leq 2</math></del>	<del><math>0 \leq t_4 \leq 3</math></del>
	<del><math>-3 \leq t_3 - t_4 \leq 1</math></del>	
$C_2 : P_1 + P_2$	<del><math>0 \leq t_3 \leq 1</math></del>	<del><math>0 \leq t_4 \leq 2</math></del>
	<del><math>-2 \leq t_3 - t_4 \leq 1</math></del>	
$C_3 : P_0 + P_3$	<del><math>0 \leq t_1 \leq 2</math></del>	<del><math>0 \leq t_2 \leq 3</math></del>
	<del><math>-3 &lt; t_1 - t_2 &lt; 1</math></del>	
$C_4 : P_0 + P_3$	<del><math>0 \leq t_1 \leq 1</math></del>	<del><math>0 \leq t_2 \leq 2</math></del>
	<del><math>-2 \leq t_1 - t_2 \leq 1</math></del>	
$C_5 : P_1 + P_3$	<del><math>0 \leq t_0 \leq \infty</math></del>	

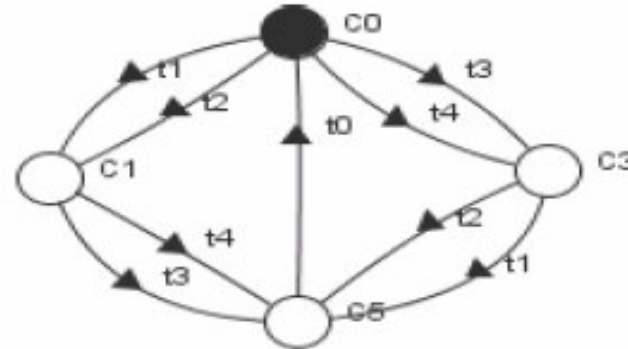
t1, t2 are in conflict



# Another example with conflicting transitions



SCG



CSCG

$$\left\{ \begin{array}{lll} C_0 : P_0 + P_2 & 0 \leq t_1 - t_2 \leq 0 & -2 \leq t_1 - t_3 \leq 2 \\ & -3 \leq t_1 - t_4 \leq 1 & -1 \leq t_2 - t_3 \leq 3 \\ & -2 \leq t_2 - t_4 \leq 2 & 0 \leq t_3 - t_4 \leq 0 \\ C_1 : P_1 + P_2 & 0 \leq t_3 - t_4 \leq 0 & \\ C_3 : P_0 + P_3 & 0 \leq t_1 - t_2 \leq 0 & \\ C_5 : P_1 + P_3 & & \end{array} \right.$$

# Contracting the state class graph

## Experimental results

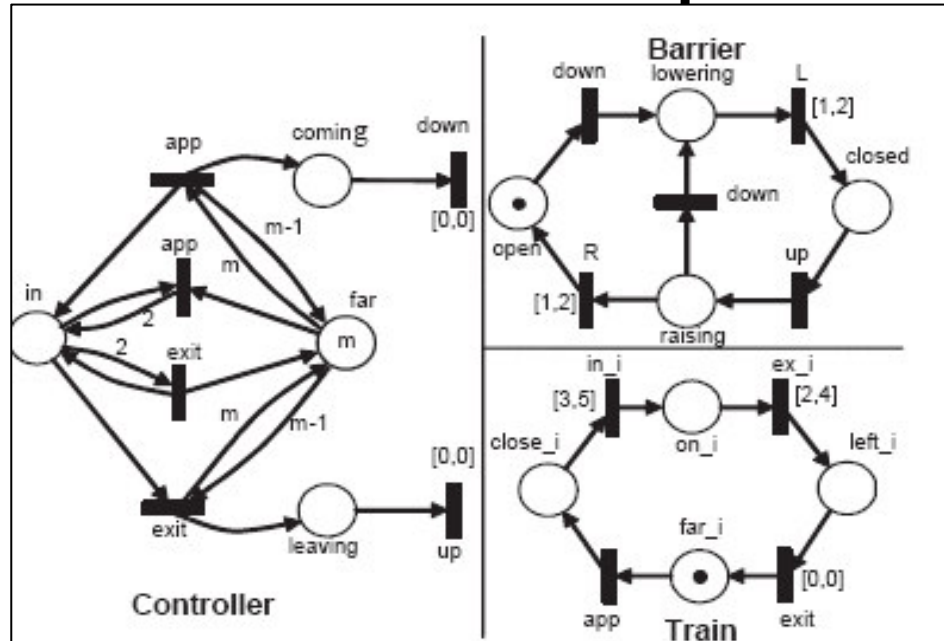


Figure 2. The level crossing models

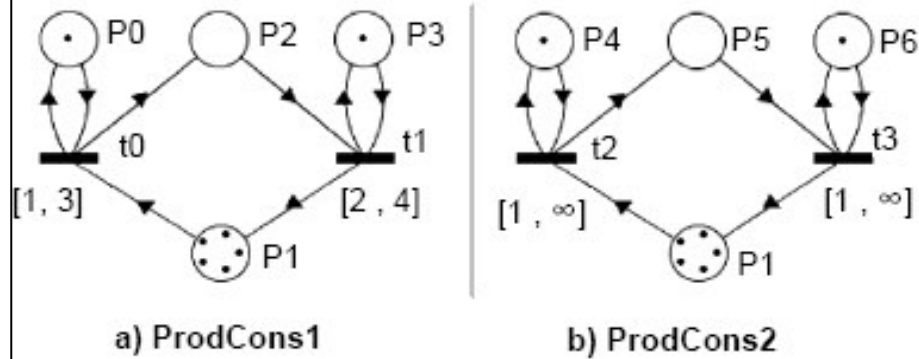


Figure 3. Producer consumer model

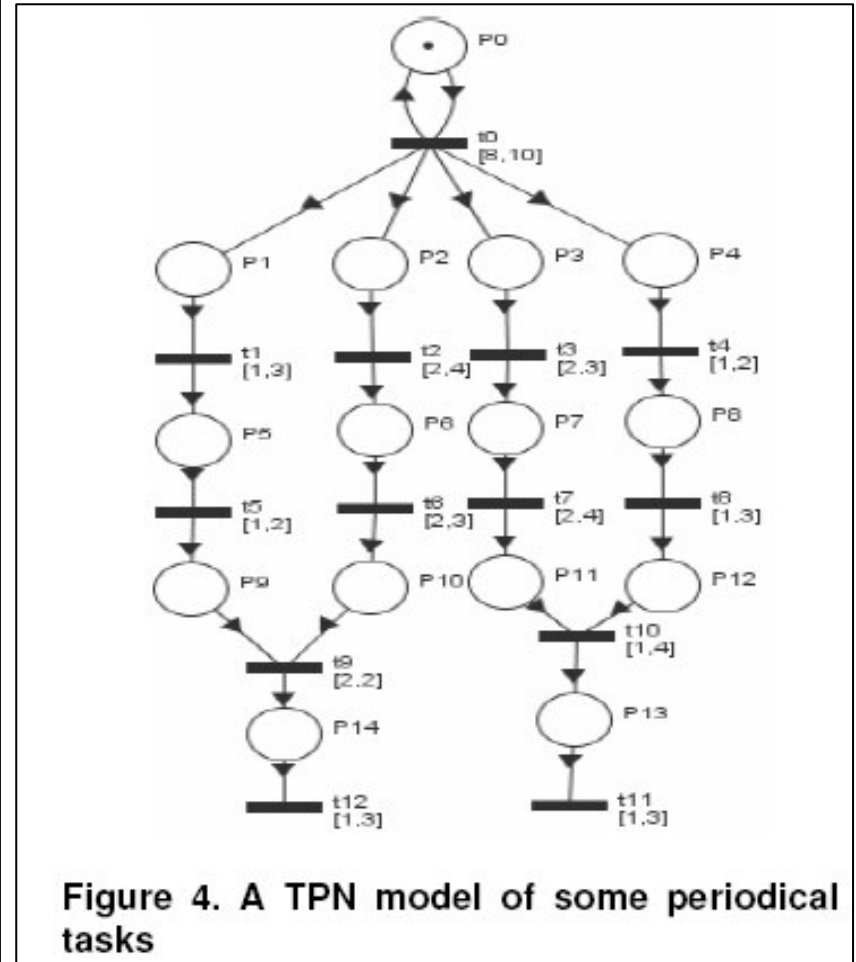


Figure 4. A TPN model of some periodical tasks

# Contracting the state class graph

## Experimental results

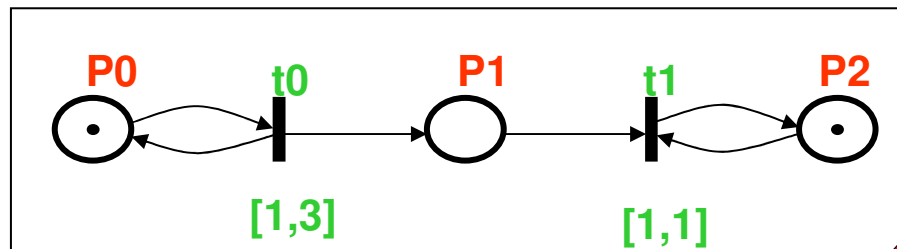
TPN	ZBG	SCG	CSCG	Ratio1
Fig.2(m=2)	147	123	113	1.09
Arcs	266	218	198	1.10
CPU(s)	0	0	0	-
Fig.2(m=3)	6299	3101	2816	1.10
Arcs	16565	7754	6941	1.12
CPU(s)	0.22	0.04	0.02	2
Fig.2(m=4)	?	134501	122289	1.10
Arcs		436896	391240	1.11
CPU(s)		4.47	2.41	1.85
Fig.3(n=2)	2941	748	519	1.44
Arcs	9952	2460	1678	1.47
CPU(s)	0.02	0.01	0	-
Fig.3(n=3)	100060	4604	2834	1.62
Arcs	485732	21891	13208	1.66
CPU(s)	20.57	0.09	0.04	2.25
Fig.3(n=4)	?	14086	8159	1.73
Arcs		83375	47592	1.75
CPU(s)		1.41	0.59	2.39
Fig.3(n=5)	?	31657	17643	1.79
Arcs		217423	120804	1.80
CPU(s)		8.77	3.22	2.72
Fig.3(n=6)	?	77208	37876	2.04
Arcs		624158	294363	2.12
CPU(s)		53.47	16.37	3.27
Fig.4	24015	18543	12910	1.44
Arcs	86621	65403	45970	1.42
CPU(s)	5.75	3.27	1.62	2.02

$$\text{Ratio1} = \text{SCG} / \text{CSCG}$$

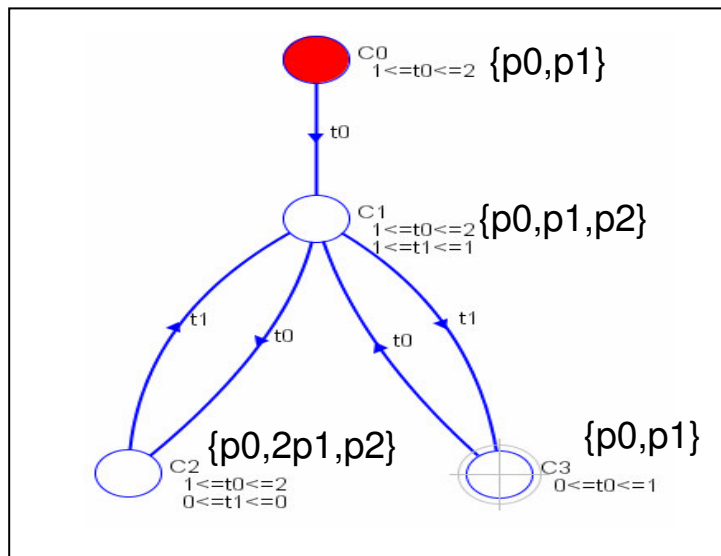
Gain in time and size grows with the size of the model

# **Model checking of SCG/CSCG using Büchi Automata method**

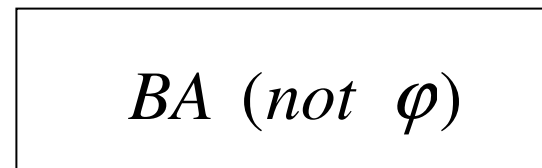
# Büchi Automata (BA) method



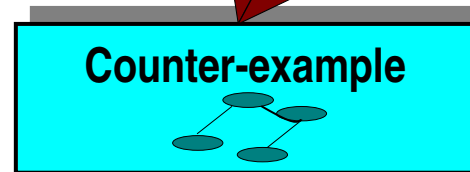
TPN model



SCG or CSCG



Negation of a property



Property not satisfied

Property satisfied

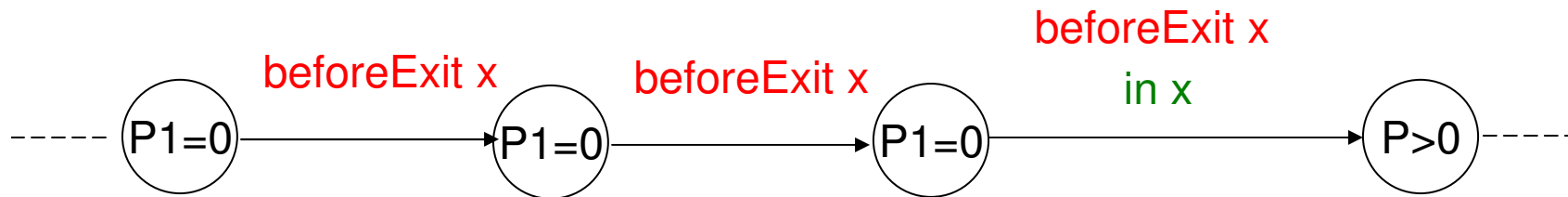
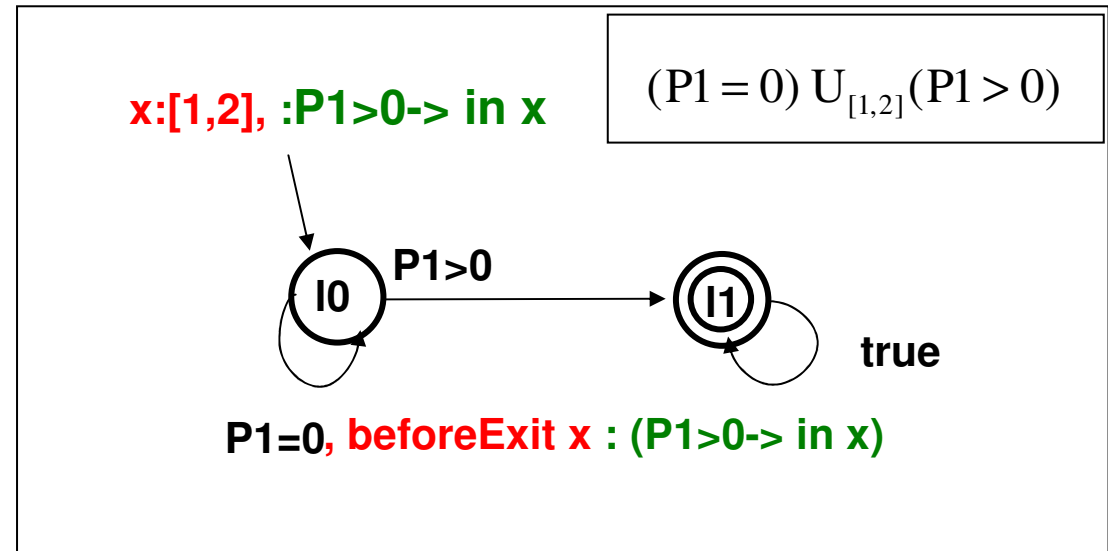
$L(TPN) \cap L(BA) \neq \emptyset$

# **Interval Timed Büchi Automata (ITBA)**

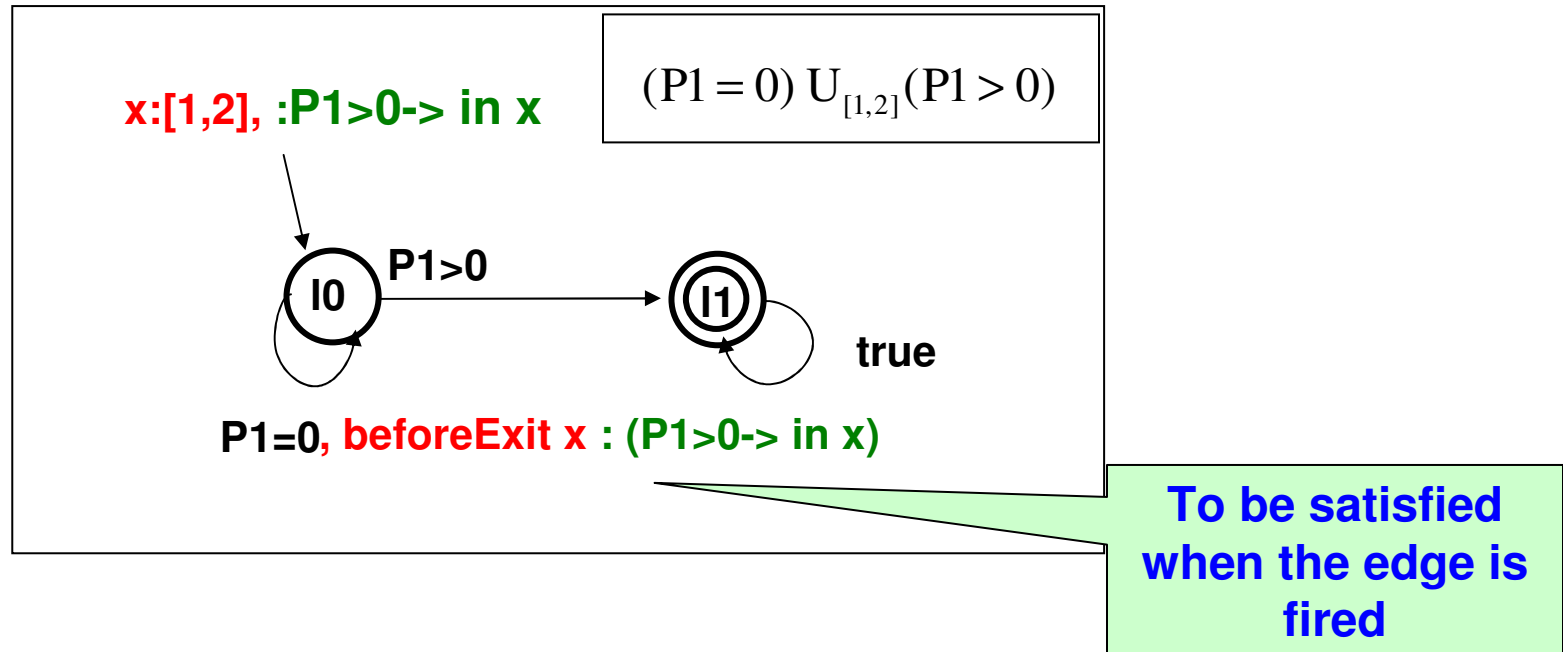
# Interval Timed Büchi Automata

BA + {interval delays} +  
 {guards on delays} +  
 {(re)setting /deactivation of delays}

Guards on delays: { before x,  
 in x,  
 beforeExit x,  
 after x,  
 true}



# Interval Timed Büchi Automata



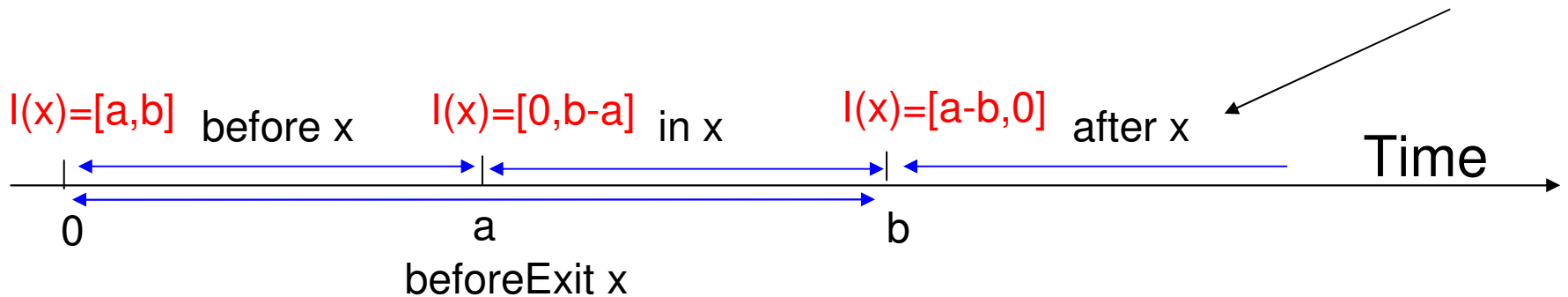
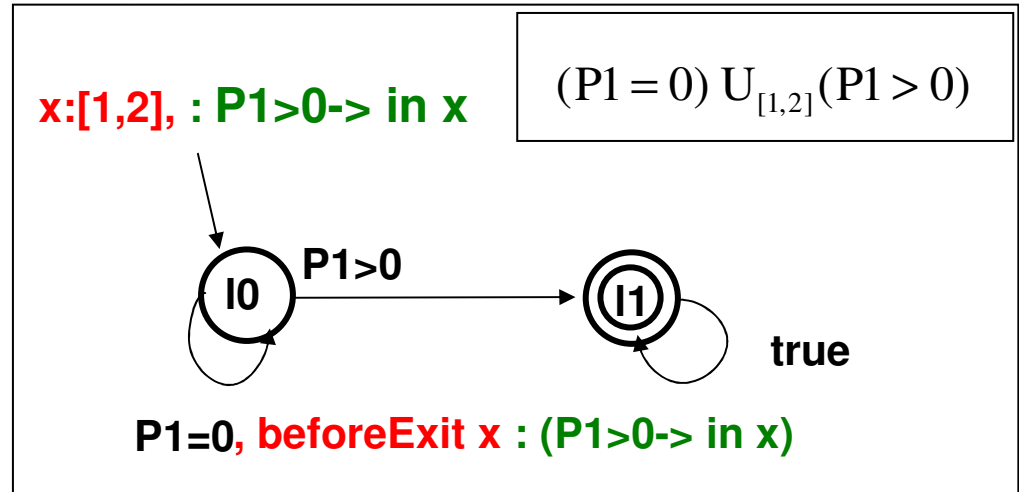
- With each delay  $x$  is associated a **dynamic interval**  $I(x)$ .
- $I(x)$  can be set to **an interval** at the beginning or when an edge is fired.
- When  $I(x)$  is set to **an interval**, its **bounds decrease** with **time** until it is **deactivated**.

**State:**  $(I_i, A, I)$



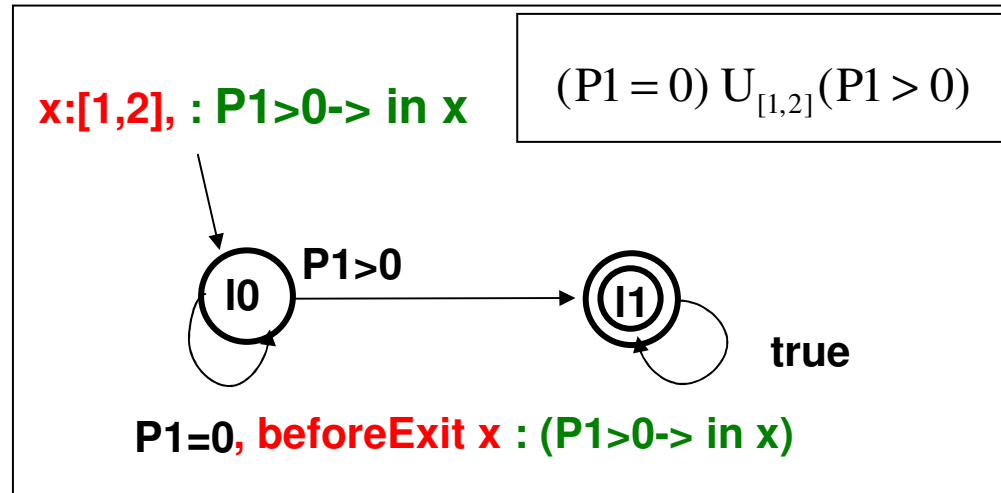
# Interval Timed Büchi Automata

State:  $(l_j, A, l)$



before x	in x	beforeExit x	after x
$\downarrow I(x) > 0$	$\downarrow I(x) \leq 0, \uparrow I(x) \geq 0$	$\uparrow I(x) \geq 0$	$\uparrow I(x) < 0$

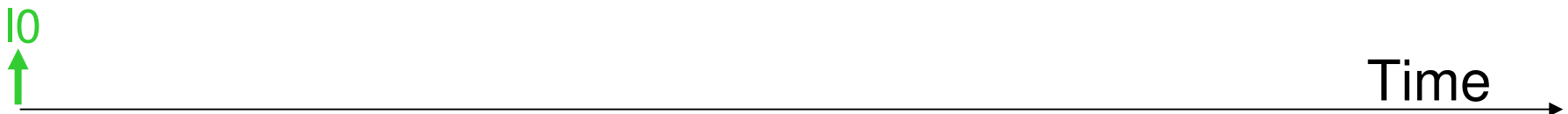
# Interval Timed Büchi Automata



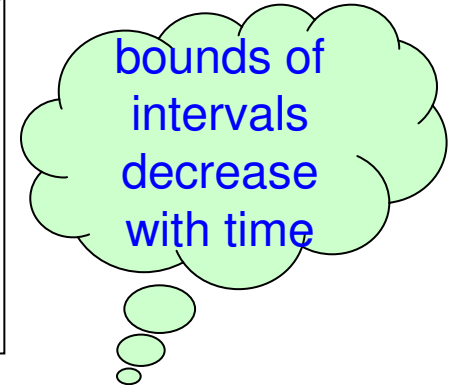
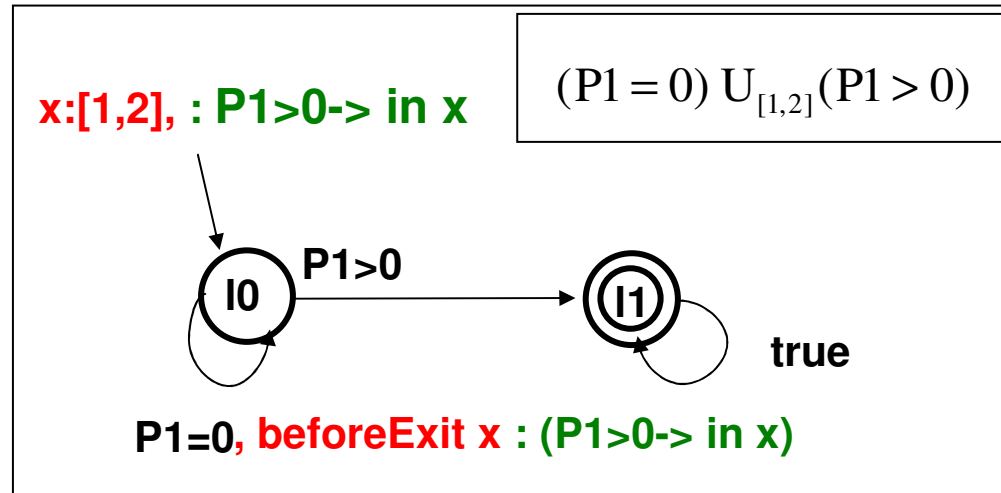
State  $q = (I_0, A, I), A = \{x\}, I(x) = [1,2]$



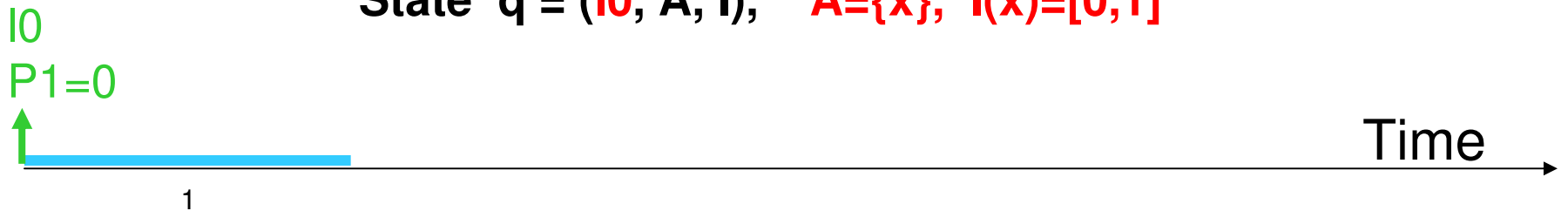
State  $q = (I_0, A, I), A = \{x\}, I(x) = [1,2]$



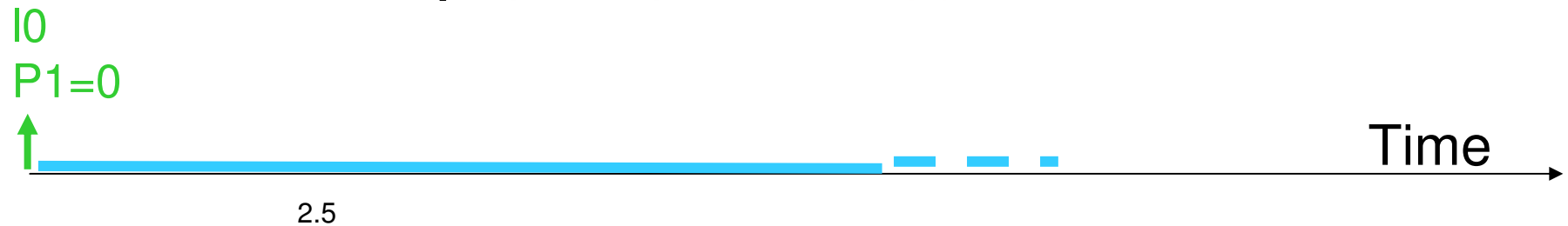
# Interval Timed Büchi Automata



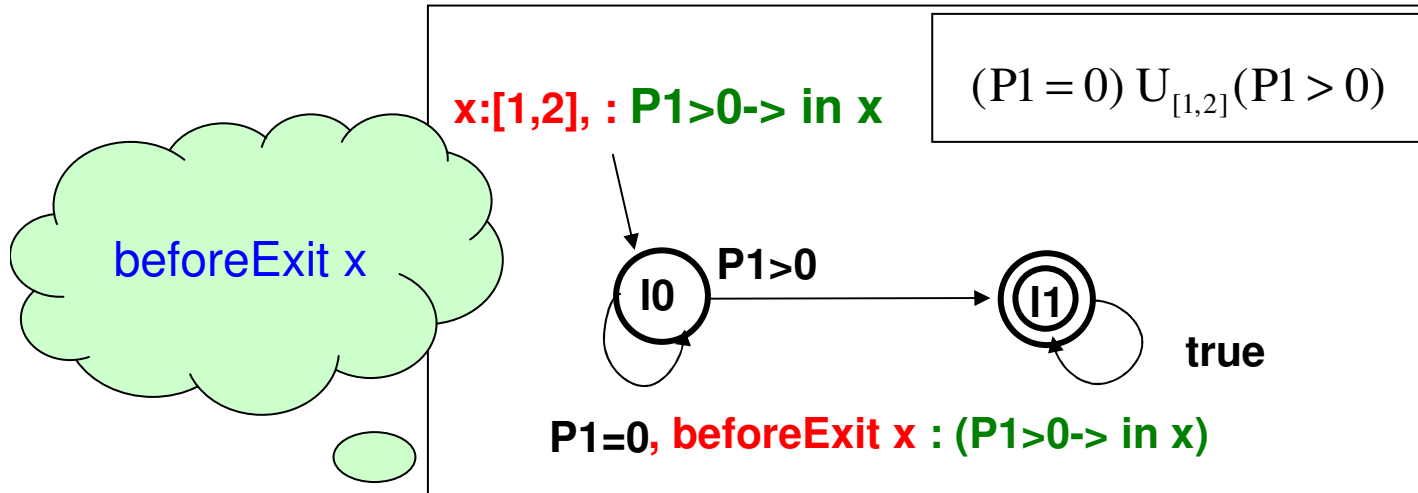
State  $q = (I0, A, I)$ ,  $A=\{x\}$ ,  $I(x)=[0,1]$



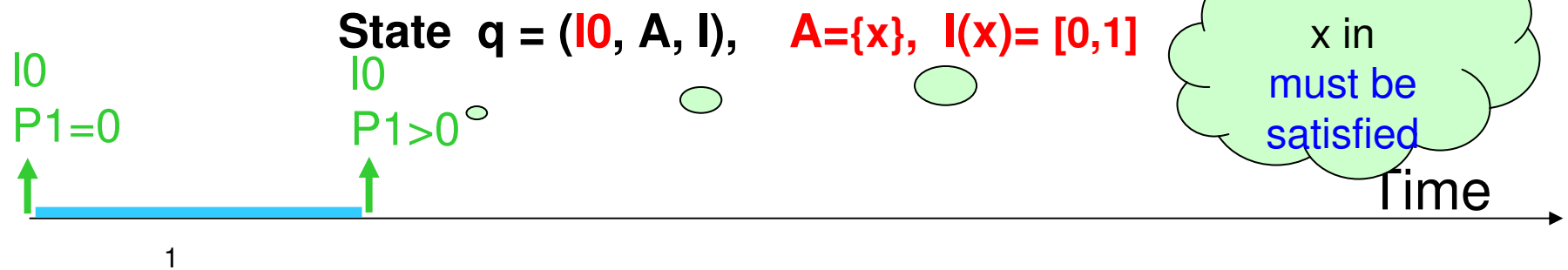
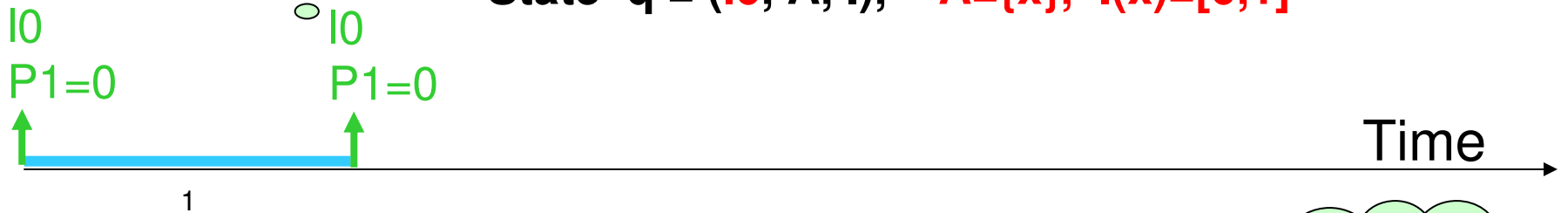
State  $q = (I0, A, I)$ ,  $A=\{x\}$ ,  $I(x)=[-1.5,-0.5]$



# Interval Timed Büchi Automata

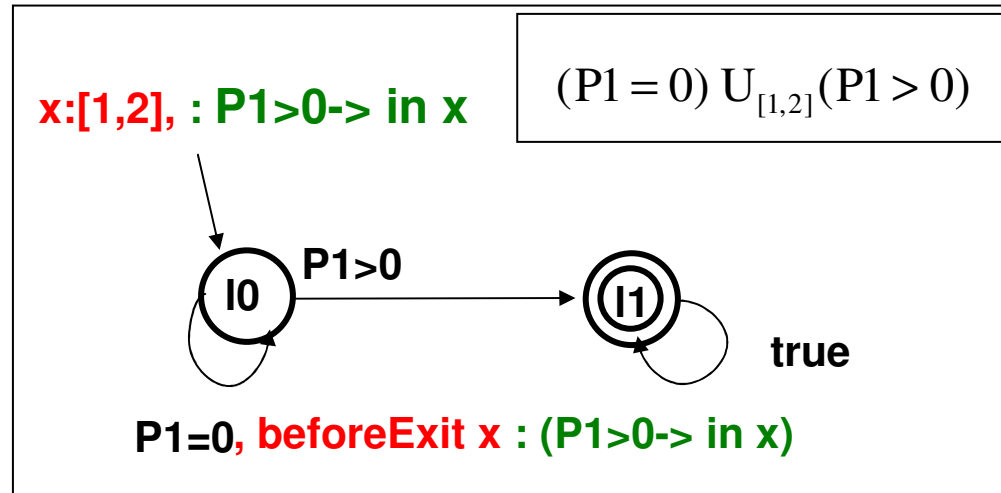


State  $q = (I_0, A, I), A=\{x\}, I(x)=[0,1]$



State  $q = (I_0, A, I), A=\{x\}, I(x)=[0,1]$

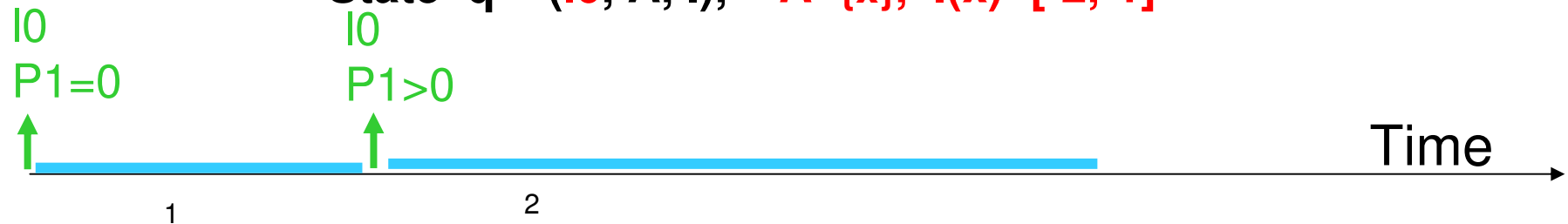
# Interval Timed Büchi Automata



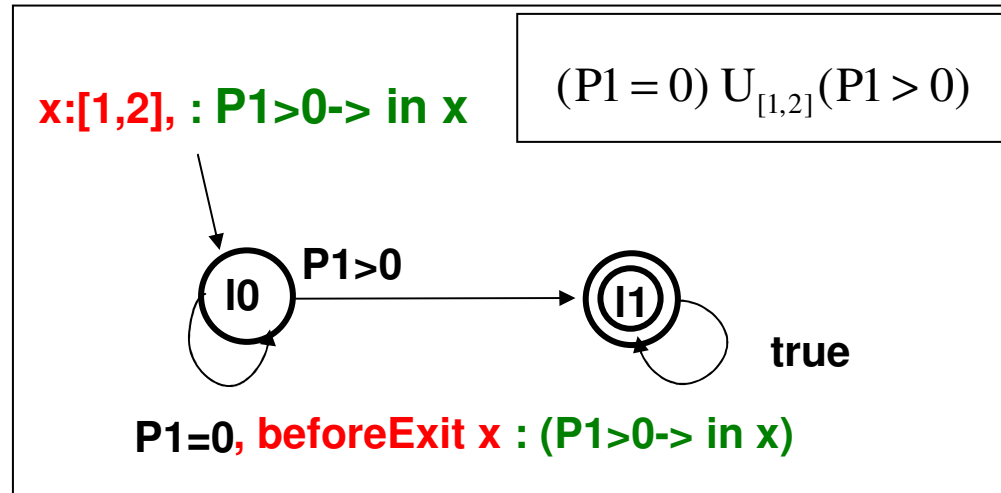
State  $q = (I0, A, I), A=\{x\}, I(x)=[-1,0]$



State  $q = (I0, A, I), A=\{x\}, I(x)=[-2,-1]$

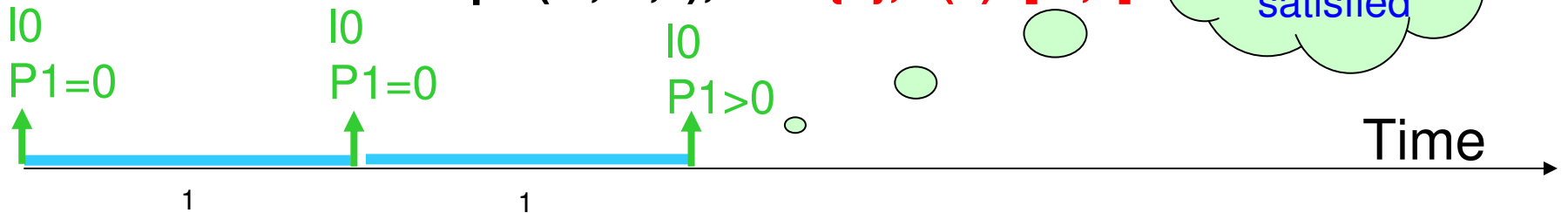


# Interval Timed Büchi Automata



$P1 > 0 \rightarrow$   
 $x \text{ in}$   
 must be  
 satisfied

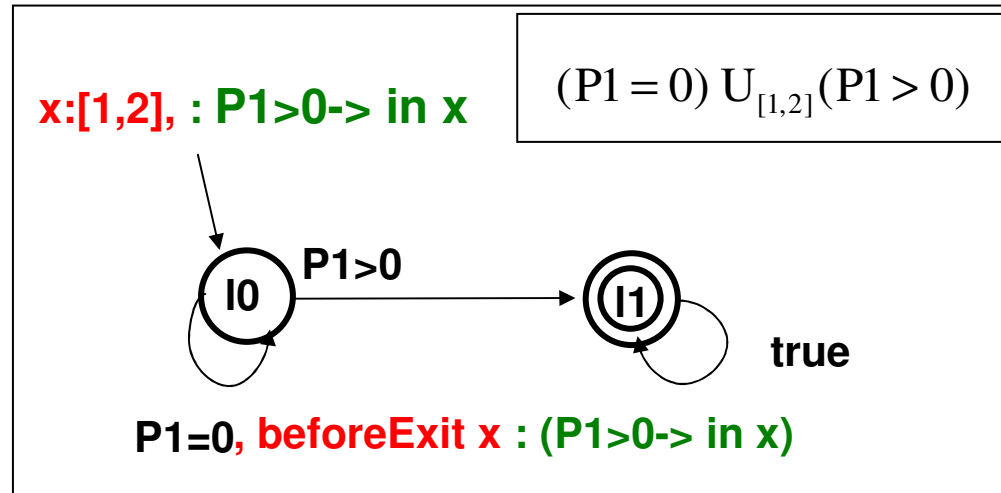
State  $q = (I0, A, I), A = \{x\}, I(x) = [-1, 0]$



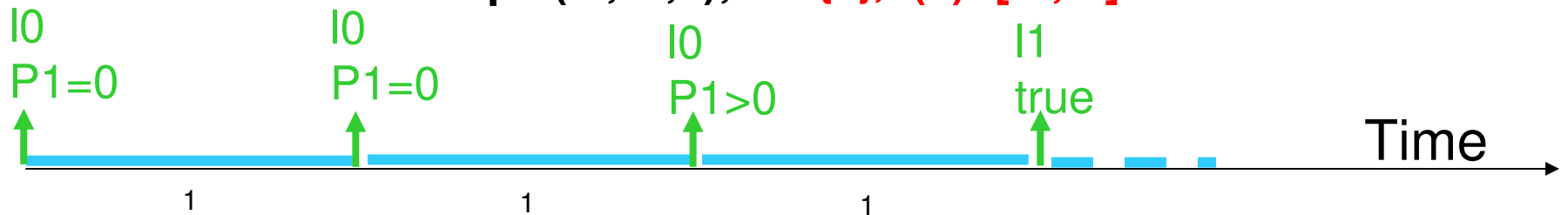
State  $q = (I1, A, I), A = \{x\}, I(x) = [-2, -1]$



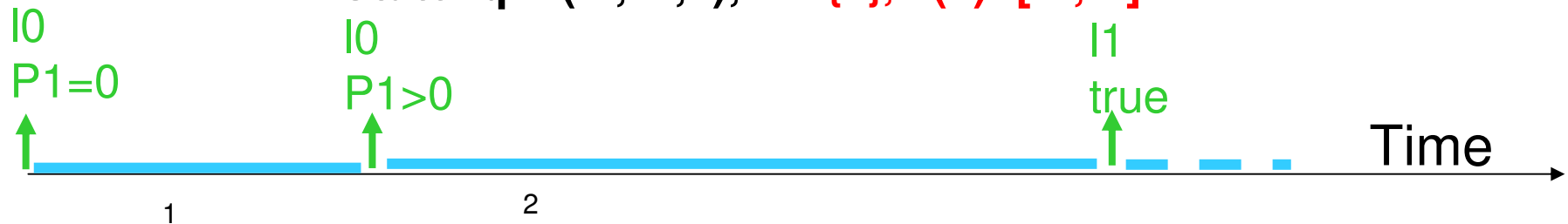
# Interval Timed Büchi Automata



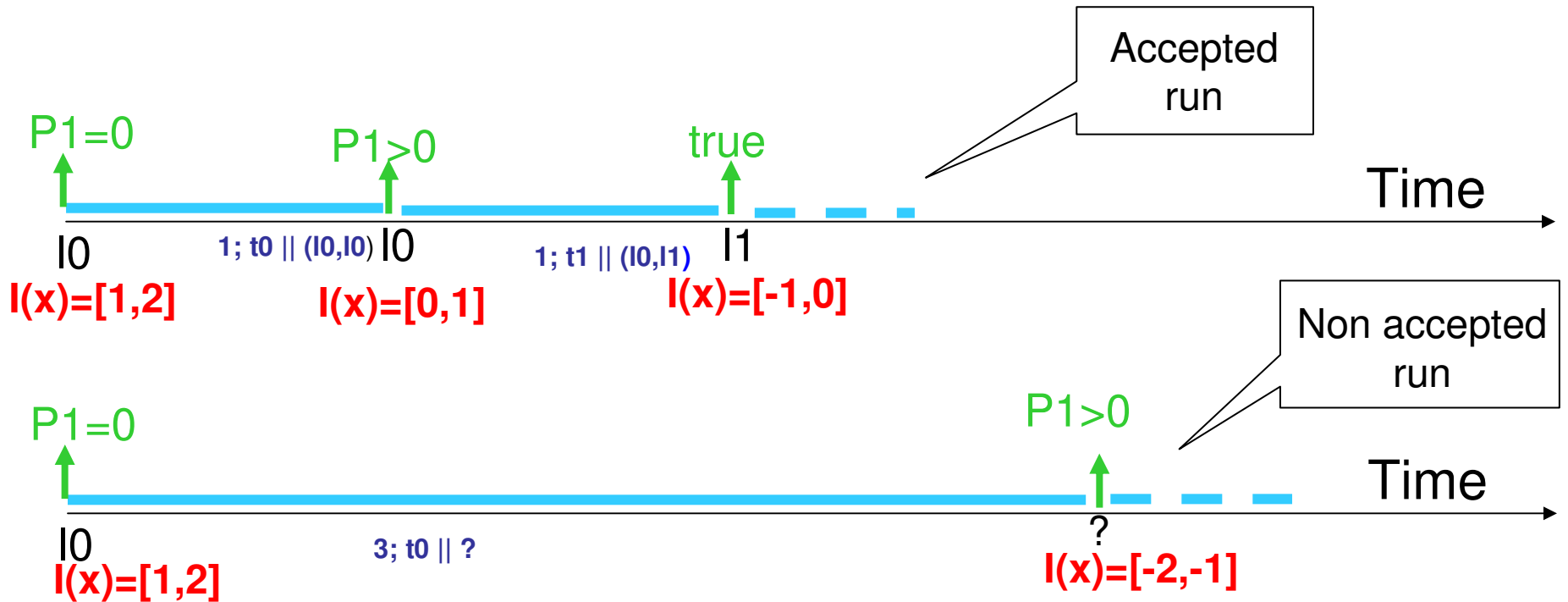
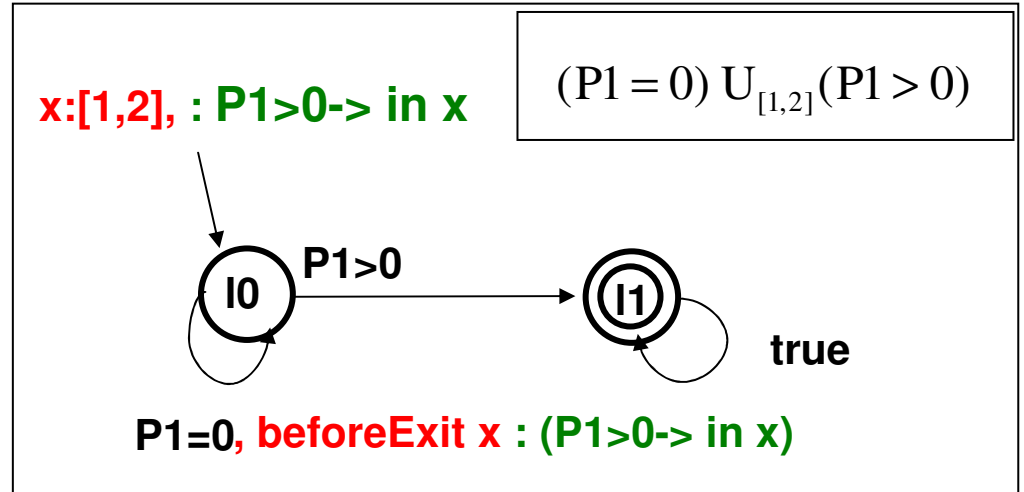
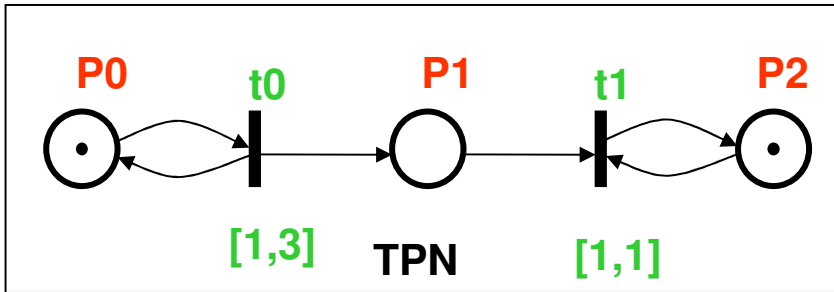
State  $q = (I1, A, I), A = \{x\}, I(x) = [-2, -1]$



State  $q = (I1, A, I), A = \{x\}, I(x) = [-2, -1]$

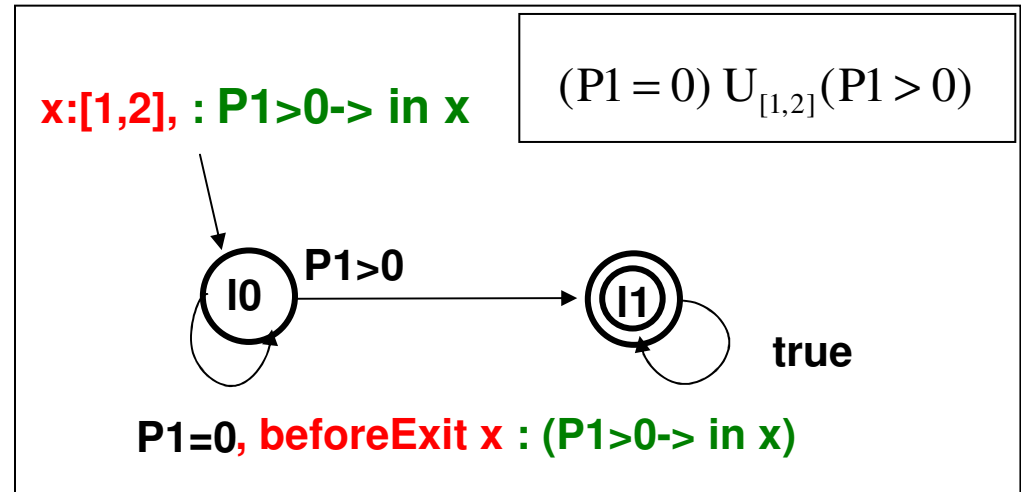
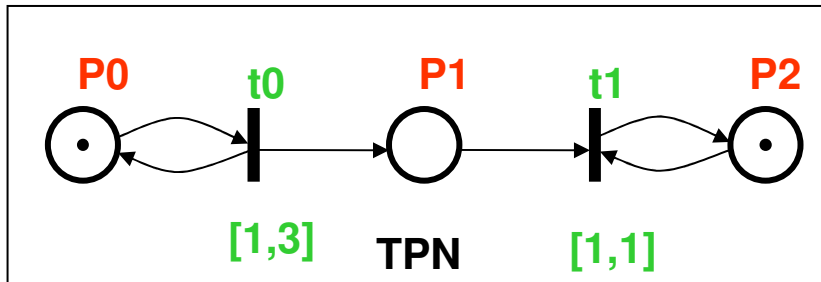


# Synchronous product $TPN \otimes ITBA$





# SCG / CSCG of $TPN \otimes ITBA$



State class of  $TPN \otimes ITBA$ :

$(M, I_p, A, F)$

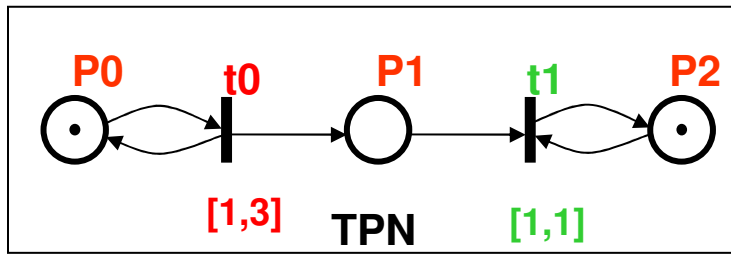
$En(M) \cup A$

Firing some transition  $t_f$ :

<i>before x</i>	<i>in x</i>	<i>beforeExit x</i>	<i>after x</i>
$x - t_f > 0$	$x - t_f = 0$	$x - t_f \geq 0$	$x - t_f < 0$

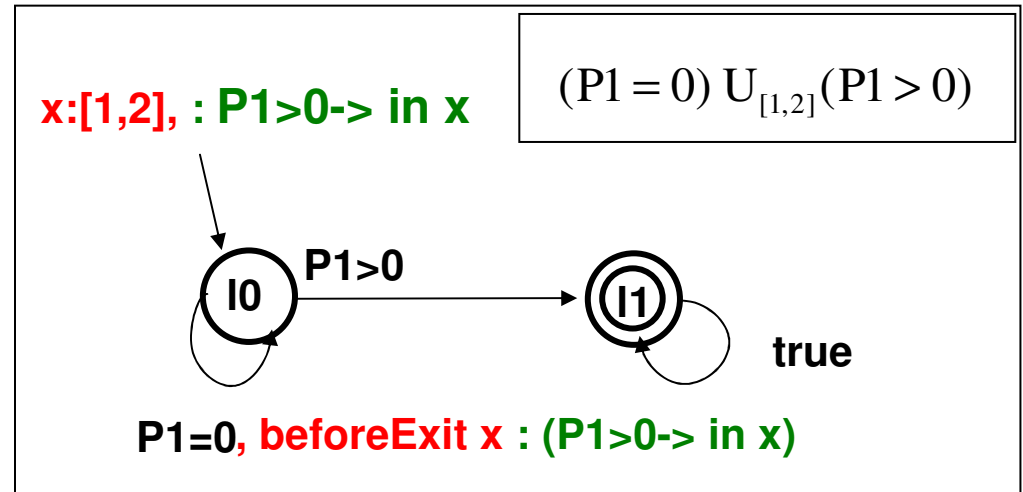
*before x* is satisfied when  $t_f$  is fired

# TPN ⊗ ITBA



C0

$(\{P_0, P_2\}, I_0), \{x\},$   
 $1 \leq x \leq 2 \wedge$   
 $1 \leq t_0 \leq 3$

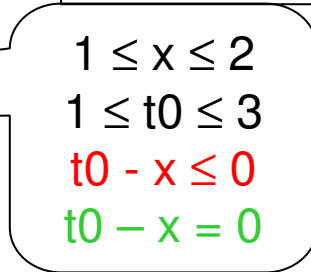
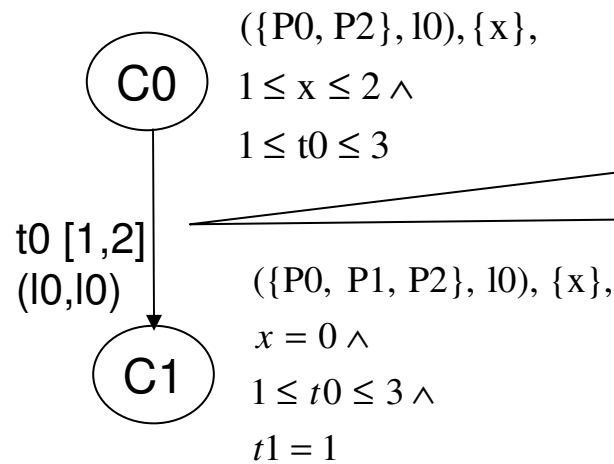
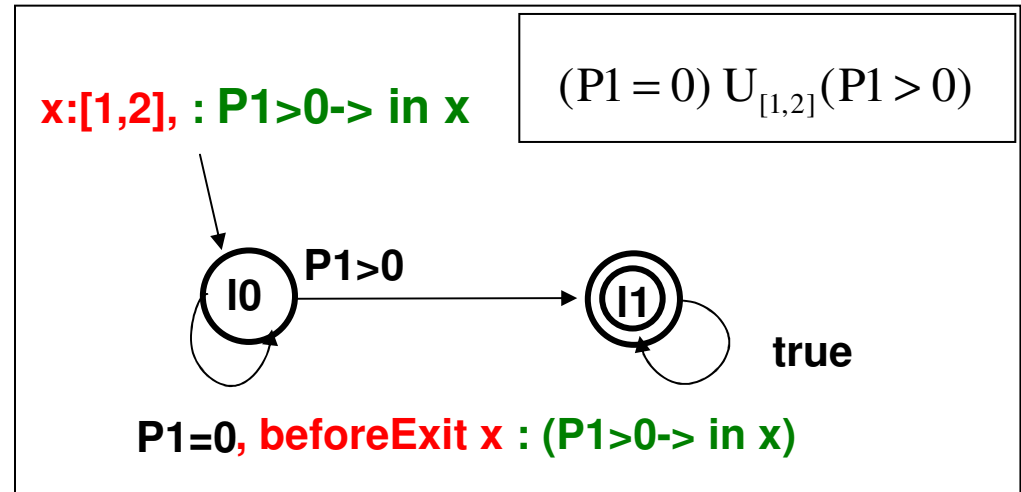
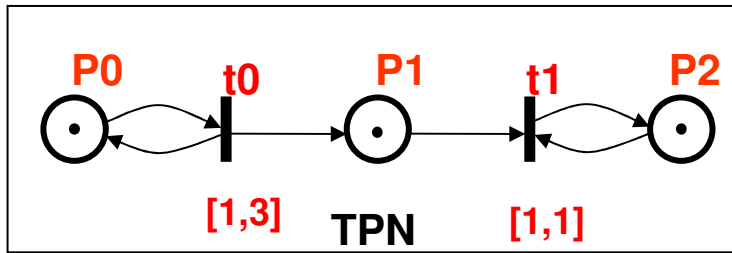


$x:[1,2], : P_1 > 0 \rightarrow \text{in } x$

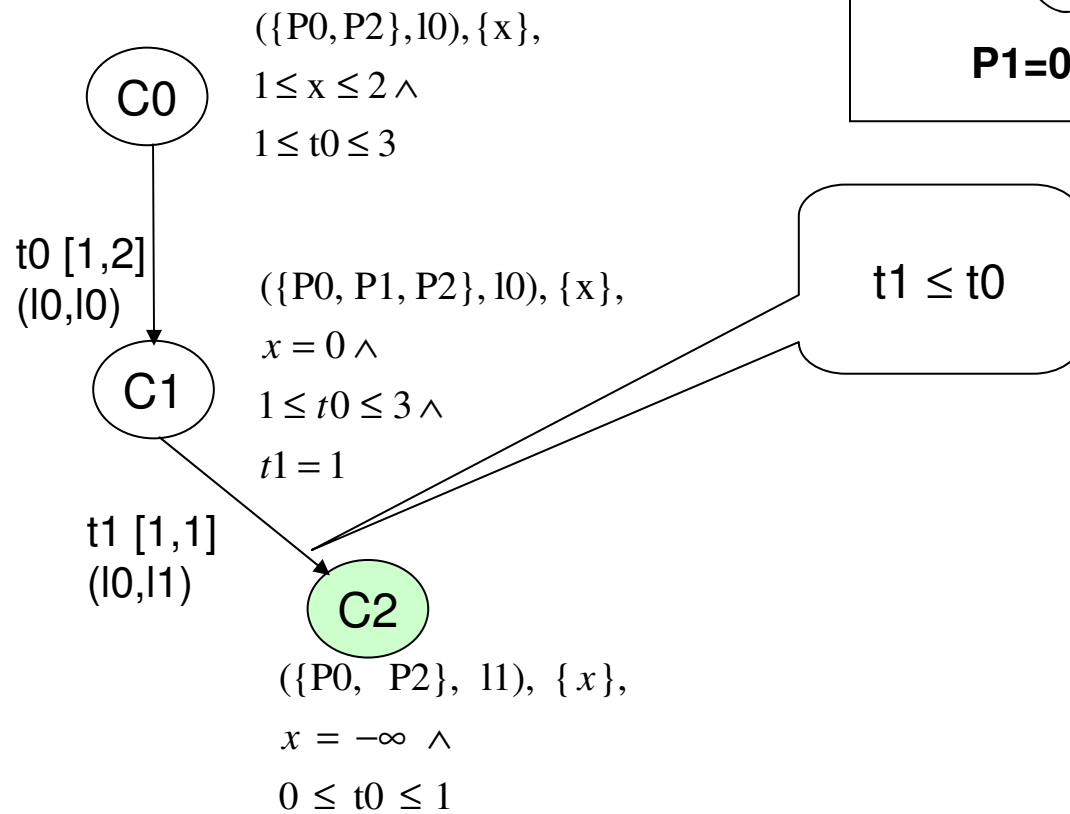
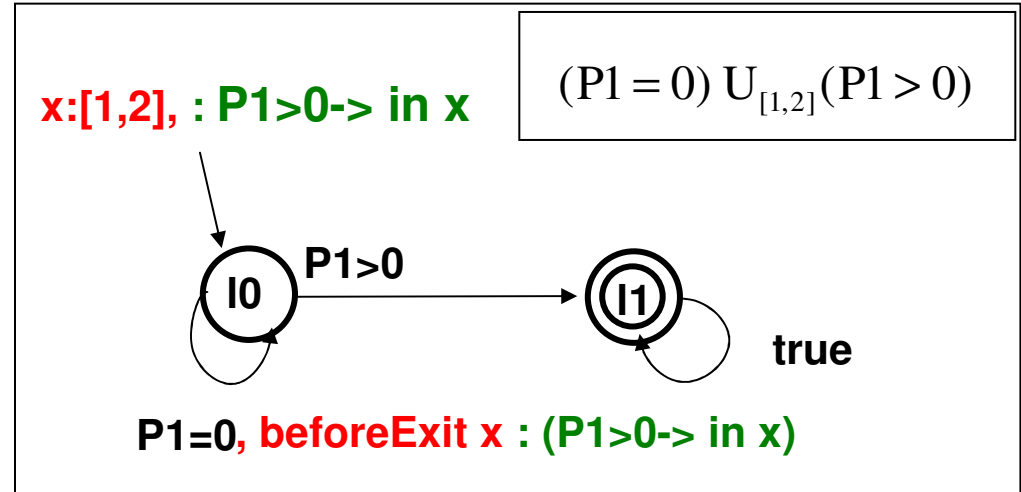
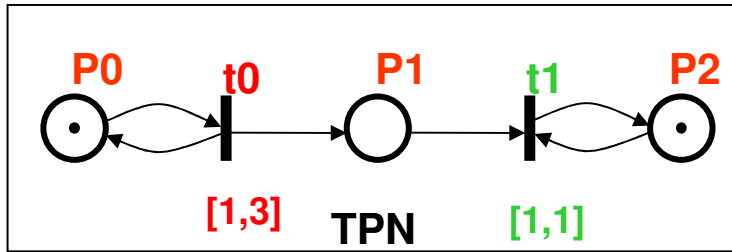
$(P_1 = 0) U_{[1,2]}(P_1 > 0)$

$P_1=0, \text{beforeExit } x : (P_1 > 0 \rightarrow \text{in } x)$

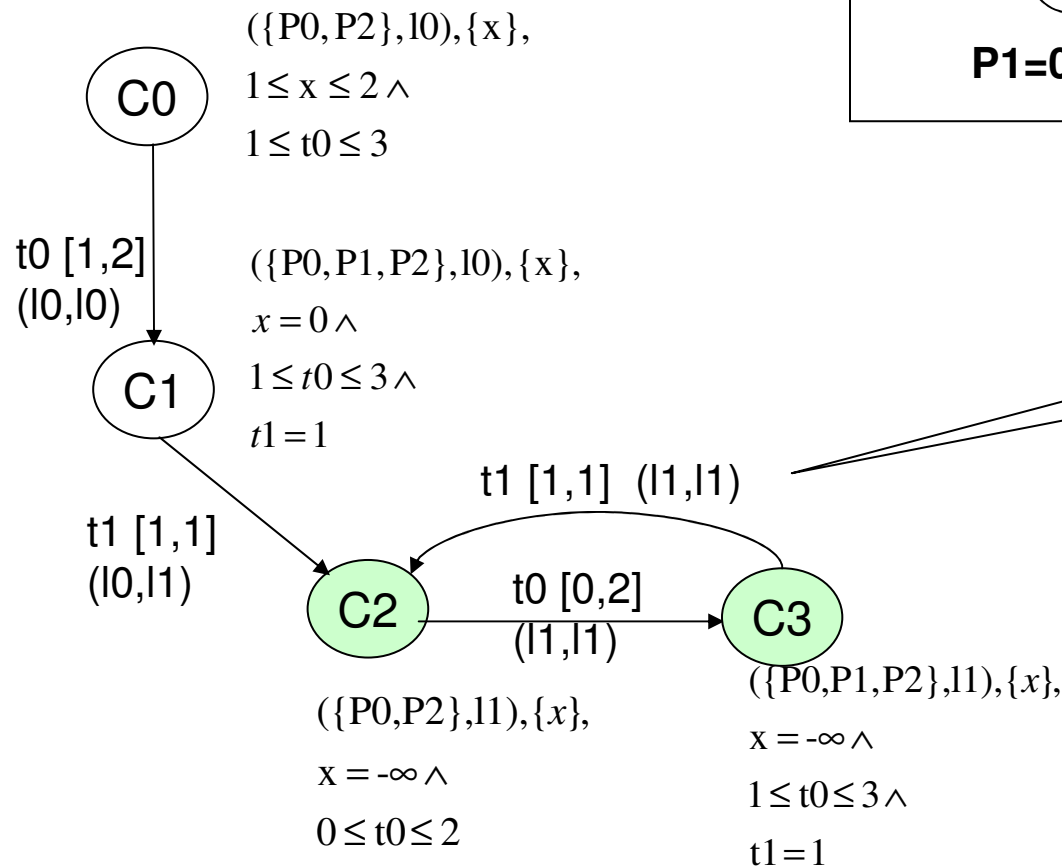
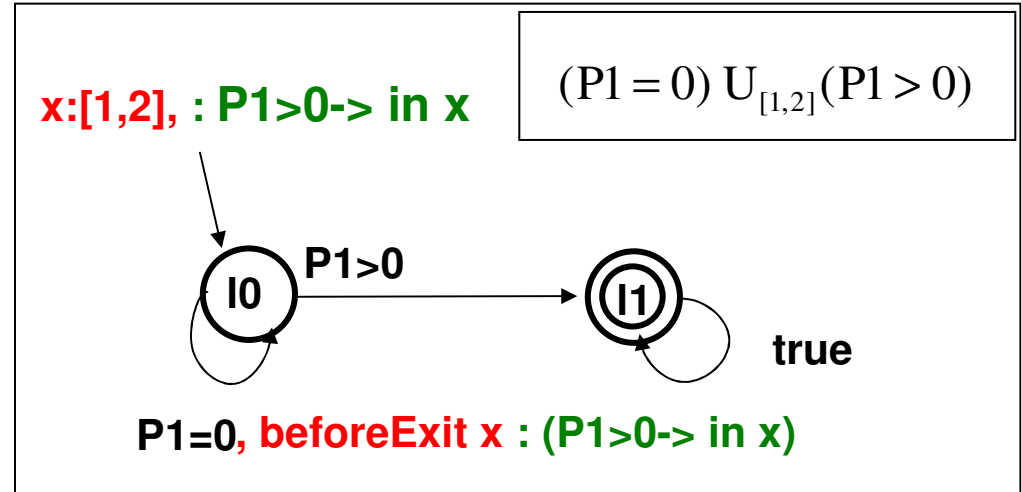
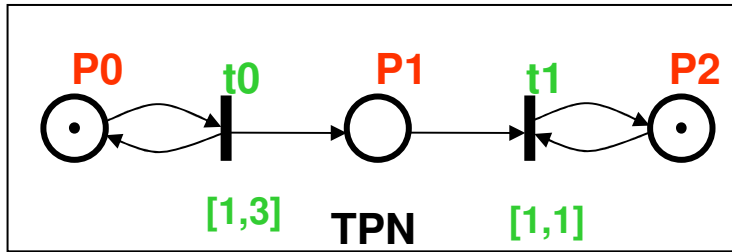
# TPN ⊗ ITBA



# TPN ⊗ ITBA



# TPN ⊗ ITBA



$L(\text{TPN}) \cap L(\text{ITBA}) \neq \emptyset$

## Conclusion

- CSCG
- An Interval Timed Büchi Automata (ITBA)
- A verification technique using ITBA and SCG / CSCG

**Merci!**