Timed ATL:

Forget Memory, Just Count

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a joint work with
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Outline

Main Contributions

- New counting strategies for Timed ATL (TATL)
- Hierarchy of semantics for different strategies of Timed ATL
- Counting strategies avoiding tracking the passage of time have the same expressivity as timed strategies

Specification and Verification of Strategic Ability

- Many important properties are based on strategic ability
- Functionality $\approx$ ability of authorized users to complete some tasks
- Security $\approx$ inability of unauthorized users to complete certain tasks
- One can try to formalize such properties in modal logics of strategic ability, such as (T)ATL or Strategy Logic
- ...and verify them by model checking

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Motivation: Two Projects

- Project PAS - CNRS on Parametric Verification,
- Project PAS - University of Luxembourg on Verification of Voter-Verifiable Voting Protocols VoteVerif,
- Example properties: ballot confidentiality, coercion-resistance, end-to-end voter-verifiability,
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Related Work

Previous

- Alternating-time temporal logic [Alur et al. 1997-2002]
- Timed alternating-time temporal logic [Henzinger and Prabhu, LAMAS 2006]
- Model checking timed ATL for durational concurrent game structures [Laroussinie, Markey, Oreiby, LAMAS 2006]

Current

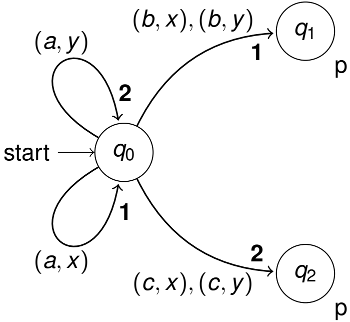
- Timed ATL: Forget Memory, Just Count [Andre, Petrucci, Jamroga, Knapik, Penczek, AAMAS 2017]

Models

A Tight Durational Concurrent Game Structure is a 7–tuple $\mathcal{A} = (\mathit{Agents}, \Sigma, \mathcal{Q}, \mathcal{AP}, \mathcal{L}, \mathit{protocol}, \mathit{trans})$, where:

- *Agents* is a finite set of all the agents,
- Σ is a finite set of actions,
- $\mathcal{Q}$ is a finite set of locations,
- $\mathcal{AP}$ is a set of atomic propositions,
- $\mathcal{L}\colon \mathcal{Q} \to \mathcal{P}(\mathcal{AP})$ is a location labeling function,
- $\mathit{protocol}\colon \mathit{Agents} \times \mathcal{Q} \to \mathcal{P}(\Sigma) \setminus \{\emptyset\}$ is a protocol function,
- $\mathit{trans}\colon \mathcal{Q} \times \Sigma^{|\mathit{Agents}|} \to \mathcal{Q} \times \mathbb{N}_{+}$ is a transition function.

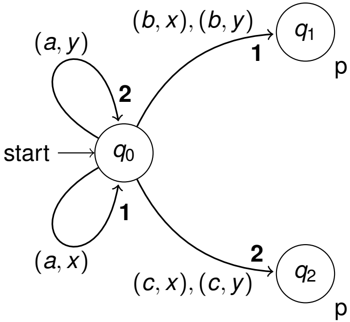
Models, cont'd



Runs are modeled in the space (locations x time) $S := Q \times \mathbb{N}$:

$$(q_0, 0) \xrightarrow{(a,y)} (q_0, 2) \xrightarrow{(a,x)} (q_0, 3) \xrightarrow{(c,y)} (q_2, 5)$$

Models, cont'd



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Notations

Let $q \in \mathcal{Q}$, $s \in \mathcal{S}$ and $\pi \in \mathcal{S}^+ \cup \mathcal{S}^\omega$.

- $loc(s)$ and $time(s)$: the location and time of s, resp.,
- $\pi(i)$: the i–th state of π,
- π_i: the prefix of π of length i,
- π^i: the suffix of π starting from $\pi(i)$,

Notations

- if π is finite:
  - π_F: its final state,
  - $\#_F(\pi)$: the number of states of π whose location is $loc(\pi_F)$.

Example. Count how many times the final location appears along π, e.g.:

$$\begin{aligned}\pi &= \big((q_0, 0), (q_0, 2)\big),\\ \pi' &= \big((q_0, 0), (q_0, 2), (q_0, 3)\big),\\ \pi'' &= \big((q_0, 0), (q_0, 2), (q_0, 3), (q_2, 5)\big),\end{aligned}$$

$\#_F(\pi) = 2$, $\#_F(\pi') = 3$, $\#_F(\pi'') = 1$.

Standard Strategies, cont'd

Let $a \in Agents$:

Timed perfect recall strategies (Σ_T)

Functions $\sigma_a \colon \mathcal{S}^+ \to \Sigma$ s.t., $\forall_{\pi \in \mathcal{S}^+} \sigma_a(\pi) \in protocol_a(loc(\pi_F))$.

(Intuition: no constraints, apart from the protocol of agent *a*)

Timed memoryless strategies (Σ_t)

Strategies $\sigma_a \in \Sigma_T$ s.t., for each $\pi, \pi' \in \mathcal{S}^+$, if $\pi_F = \pi'_F$, then $\sigma_a(\pi) = \sigma_a(\pi')$.

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New Strategies

Counting strategies ($\Sigma_{\#}$)

Strategies $\sigma_a \in \Sigma_T$ s.t. for each $\pi, \pi' \in \mathcal{S}^+$, if $loc(\pi_F) = loc(\pi'_F)$ and $\#_F(\pi) = \#_F(\pi')$, then $\sigma_a(\pi) = \sigma_a(\pi')$.

(Intuition: action selection depends on the number of visits to the location of π_F)

Alternative notation

A counting strategy is a function $\sigma_a^{\#} \colon \mathcal{Q} \times \mathbb{N} \to \Sigma$ s.t.
$\sigma_a^{\#}(q, k) := \sigma_a(\pi)$ if $q = loc(\pi_F)$ and $k = \#_F(\pi)$.

New Strategies, cont'd

Threshold strategies ($\Sigma_{\#_n}$)

A counting strategy $\sigma_a^{\#} \in \Sigma_{\#}$ is called *n*–**threshold** for some $n \in \mathbb{N}_+$ iff for each location $q \in \mathcal{Q}$ there exist:

- actions $\text{act}_1, \ldots, \text{act}_{n+1} \in \Sigma$, and
- integer intervals $I_1 = [1, i_1), I_2 = [i_1, i_2), \ldots, I_{n+1} = [i_n, \infty)$

s.t. for all $1 \leq j \leq n+1$: $\sigma_a^{\#}(q, k) = \text{act}_j$ if $k \in I_j$.

Example: a counting strategy is 2–threshold if for any location $q \in \mathcal{Q}$ there are **three** actions $\text{act}_1, \text{act}_2, \text{act}_3$ s.t. first only act_1 is used when q is visited, then only act_2, and finally only act_3, ad infinitum.

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Joint Strategies

- A joint strategy σ_A for agents $A \subseteq \mathit{Agents}$ is a tuple of strategies, one per agent $a \in A$.

  Notation: if $A = \{a_1, \ldots, a_k\}$ for some $k \in \mathbb{N}$ and $\sigma_A = (\sigma_{a_1}, \ldots, \sigma_{a_k})$ is a joint strategy for A, then for each $i \in \mathbb{N}$ and $\pi \in \mathcal{S}^\omega$ denote $\sigma_A(\pi_i) := (\sigma_{a_1}(\pi_i), \ldots, \sigma_{a_k}(\pi_i))$.

- The outcome of σ_A in state $s \in \mathcal{S}$ is the set $\mathit{out}(s, \sigma_A) \subseteq \mathcal{S}^\omega$ s.t. $\pi \in \mathit{out}(s, \sigma_A)$ iff $\pi(0) = s$ and for each $i \in \mathbb{N}$ $\pi(i) \xrightarrow{(\sigma_A(\pi_i), \mathrm{act}')} \pi(i+1)$ for some $\mathrm{act}' \in \mathit{protocol}_{\overline{A}}(\mathit{loc}(\pi(i)))$.

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Syntax of TATL

Timed Alternating-Time Temporal Logic (TATL)

The language of TATL is defined by the following grammar:

$$\phi ::= \mathsf{p} \mid \neg\phi \mid \phi \vee \phi \mid \langle\!\langle A \rangle\!\rangle X\phi \mid \langle\!\langle A \rangle\!\rangle \phi U_{\sim\eta}\phi \mid \langle\!\langle A \rangle\!\rangle \phi R_{\sim\eta}\phi,$$

where $\mathsf{p} \in \mathcal{AP}$, $A \subseteq Agents$, $\sim \in \{\leq, =, \geq\}$, and $\eta \in \mathbb{N}$.

We interpret $\langle\!\langle A \rangle\!\rangle\psi$ as "the coalition A has a strategy to enforce ψ", X stands for "in the next state", U for "until", and R for "release".

Derived modalities: F ("in the future") and G ("globally"): $\langle\!\langle A \rangle\!\rangle F_{\sim\eta}\phi := \langle\!\langle A \rangle\!\rangle \top U_{\sim\eta}\phi$, $\langle\!\langle A \rangle\!\rangle G_{\sim\eta}\phi := \langle\!\langle A \rangle\!\rangle \bot R_{\sim\eta}\phi$.

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$$\phi ::= \mathsf{p} \mid \neg\phi \mid \phi \vee \phi \mid \langle\langle A\rangle\rangle X\phi \mid \langle\langle A\rangle\rangle \phi U_{\sim\eta}\phi \mid \langle\langle A\rangle\rangle \phi R_{\sim\eta}\phi,$$

where $\mathsf{p} \in \mathcal{AP}$, $A \subseteq Agents$, $\sim \in \{\leq, =, \geq\}$, and $\eta \in \mathbb{N}$.

We interpret $\langle\langle A\rangle\rangle\psi$ as "the coalition A has a strategy to enforce ψ", X stands for "in the next state", U for "until", and R for "release".

Derived modalities: F ("in the future") and G ("globally"): $\langle\langle A\rangle\rangle F_{\sim\eta}\phi := \langle\langle A\rangle\rangle \top U_{\sim\eta}\phi$, $\langle\langle A\rangle\rangle G_{\sim\eta}\phi := \langle\langle A\rangle\rangle \bot R_{\sim\eta}\phi$.

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TATL, cont'd

$\text{TATL}_{\leq,\geq}$: a subset of TATL with only $\leq, \geq$ allowed,
e.g., $\langle\langle A \rangle\rangle G_{\geq 42}\text{safe} \in \text{TATL}_{\leq,\geq}$, $\langle\langle A \rangle\rangle F_{=13}\text{finish} \notin \text{TATL}_{\leq,\geq}$.

Examples of properties:

- $\langle\langle A \rangle\rangle G_{\geq 42}\text{safe}$: "Coalition A **has a strategy to enforce** that safe holds always after reaching 42 time units".
- $\langle\langle A \rangle\rangle F_{=13}\text{finish}$: "Coalition A **has a strategy to enforce** that finish is reached in exactly 13 time units".

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Semantics of TATL

For each type of strategy define the corresponding satisfaction relation, i.e., $\models_Y$ corresponds to Σ_Y, for $Y \in \{T, t, R, r, \#, \#_n\}$.

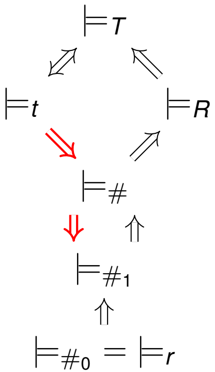
Satisfaction relation

$M, q \models_Y \langle\langle A \rangle\rangle \psi$ iff there exists a strategy $\sigma_A \in \Sigma_Y$ for A s.t. ψ holds **along each outcome** $\pi \in out((q, 0), \sigma_A)$.

Satisfaction relation over outcomes

- $\pi \models X\phi$ iff $loc(\pi(1)) \models \phi$,
- $\pi \models \phi U_{\sim\eta}\psi$ iff $loc(\pi(i)) \models \psi$ for some i s.t. $time(\pi(i)) \sim \eta$ and $loc(\pi(j)) \models \phi$ for all $j < i$,
- $\pi \models \phi R_{\sim\eta}\psi$ iff for all i: $time(\pi(i)) \sim \eta \Longrightarrow loc(\pi(i)) \models \psi$ or $loc(\pi(j)) \models \phi$ for some $j < i$,

Hierarchy of satisfaction relations



The Red implications hold only for $\text{TATL}_{\leq,\geq}$.

Key implications: timed strategies and memory

Theorem (1) Timed strategies do not need memory

For each $q \in \mathcal{Q}$ and $\phi \in$ TATL we have $q \models_T \phi$ iff $q \models_t \phi$.
(so we omit subscript in this case and write $\models$)

Lemma. Time limit

Let $\langle\langle A\rangle\rangle\psi \in$ TATL and $c \in \mathbb{N}$ be the greatest integer in ψ.
If $\sigma_A \in \Sigma_T$ implements $\langle\langle A\rangle\rangle\psi$, then there exists its reduction σ'_A s.t. $\forall_{q\in\mathcal{Q}}\forall_{t\geq c}\ \sigma'_A(q,t) = \sigma'_A(q,c+1)$, which also implements $\langle\langle A\rangle\rangle\psi$.
Intuitively, there is no need to track time after it exceeds c.

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Intuitively, there is no need to track time after it exceeds c.

Key implications: time versus order

Theorem (2) $\models_{\#} \implies \models$

For each $q \in Q$ and $\phi \in$ TATL, if $q \models_{\#} \phi$, then $q \models \phi$.

Key implications: time versus order, cont'd

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Theorem (3) $\models \implies \models_{\#}$

For each $q \in \mathcal{Q}$ and $\phi \in \text{TATL}_{\leq,\geq}$, if $q \models \phi$, then $q \models_{\#} \phi$.
(Just count locations, do not look at clock.)

(3) cannot be extended to TATL, see next slide.

Key implications: time versus order, cont'd

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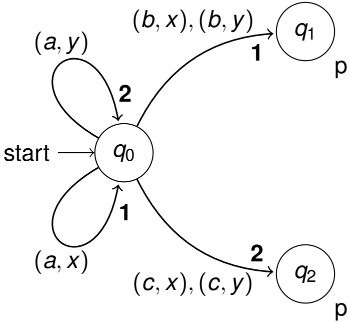
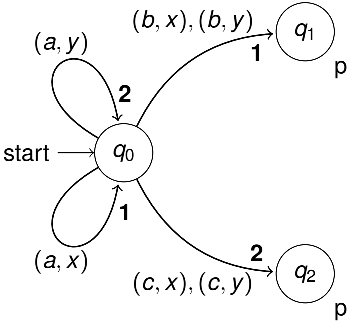
Key implications: time versus order, cot'd



$q_0 \models_T \langle\langle 1 \rangle\rangle F_{=5}$p, but $q_0 \not\models_\# \langle\langle 1 \rangle\rangle F_{=5}$p, as there is no counting strategy that allows to decide when to leave q_0 for a location labeled with p and which branch to take in order to reach the target in 5 time units.

Key implications: time versus order, cot'd



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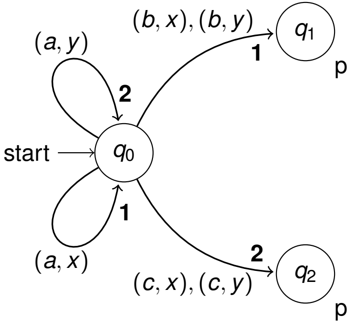

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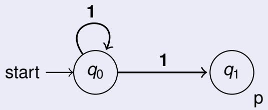
Key implications: counting up to...

Theorem (4) Threshold for $TATL_{\leq,\geq}$ is 2

For each $q \in Q$ and $\phi \in TATL_{\leq,\geq}$, if $q \models_{\#} \phi$, then $q \models_{\#_1} \phi$.

All modalities except for $U_{\geq\eta}$ need only one action, and $U_{\geq\eta}$ needs two.

...and cannot be lowered

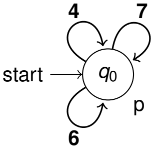


- $q_0 \models_{\#_1} \langle\langle 1 \rangle\rangle F_{\geq 5}$p: loops four times and jumps ahead
- $q_0 \not\models_{\#_0} \langle\langle 1 \rangle\rangle F_{\geq 5}$p: loops forever, or jumps too early

Key implications: counting up to. . . , ct'd

Theorem (5)

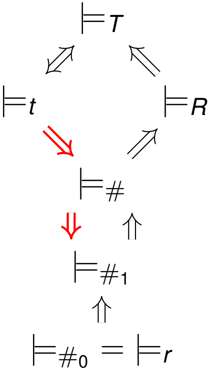
There is no threshold for TATL.



$\langle\langle 1 \rangle\rangle F_{=17}$p: **three distinct actions** needed to sum up to exactly 17 time units.

This can be extended to an arbitrary number (n) of actions using L. Mikulski's sequence: $(10)^n + 1, \ldots, (10)^n + 2^i, \ldots, (10)^n + 2^n)$ for the times of the actions.

Hierarchy of satisfaction relations



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Conclusions and Future Work

Conclusions

- Hierarchy of strategies for TATL,
- Unexpectedly, $\models$ is equivalent to $\models_{\#}$ for $\text{TATL}_{\leq,\geq}$,
- Threshold for $\text{TATL}_{\leq,\geq}$ is 2.

Future Work

- Extensions to TATL^* and parametric versions,
- Incomplete knowledge semantics,
- Model checking algorithms.

Thank you!